Southern Regional High School
Manahawkin, New Jersey

Course of Study

For

Algebra II
2308

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Southern Regional High School District
Course of Study

Department_____Mathematics_____

Course Title: **Algebra II**
Algebra II Honors
Algebra II Advanced

Essential Questions of the Course:

- How can linear and non-linear modeling be used in the analysis of real life data.
- When is it appropriate to use technology to enhance mathematical concepts?
- How can situations in physics, economics and social studies be modeled with algebraic functions?

Assessments:

- Teacher made tests and quizzes
- Projects examining real life applications
- CBL Investigations and Activities
- Graphing Calculator Investigations and Activities
- Midterm and Final Exams
- Internet Investigations

Unit of Study

Unit Title: Linear Review

Essential Questions of the Unit:

- How can linear modeling be used in the analysis of real life data?
- When is it appropriate to use technology to enhance linear modeling?

Assessments:

- Teacher made tests and quizzes
- CBL Data Collection Labs
- Project involving analysis of data collected in a lab
- Graphing Calculator Investigations

Content:

- Real number operations
- Algebraic expressions
- Linear Equations
- Linear Inequalities
- Literal Equations
- Absolute Value Equations
- Vertex
- Tables
- Graphs of Data

Skills:

- To perform real number operations.
- To evaluate algebraic expressions.
- To solve and graph linear equations.
- To solve and graph linear inequalities.
- To solve and graph absolute value equations.
- To solve value inequalities.
- To use tables and graphs to analyze data.
- To find the line of best fit on a scatter plot.
- To find the linear regression equation with the graphing calculator.

Purpose / Rationale of the Unit:

Using linear models to represent and analyze real life data enables students to see the meaning and validity of learning algebraic concepts.

New Jersey Core Curriculum Content Standards:

4.1 NUMBER AND NUMERICAL OPERATIONS

A. Number Sense – 1, 2, 3
B. Numerical Operations - 1

4.2 GEOMETRY AND MEASUREMENT

A. Geometric Properties - 3
C. Coordinate Geometry - 1
E. Measuring Geometric Objects - 2

4.3 PATTERNS AND ALGEBRA

B. Functions – 2, 2, 4
C. Modeling – 1, 2
D. Procedures – 2, 3

4.4 DATA ANALYSIS, PROBABILITY, AND DISCRETE MATHEMATICS

A. Data Analysis (Statistics) – 2, 4, 5
B. Probability

4.5 MATHEMATICAL PROCESSES

A. Problem Solving – 1-5

(For descriptive narrative, See Appendix)

The use of technology is incorporated throughout the mathematics curriculum.
Included is the use of graphing calculators and computer hardware and software to use mathematical or logical functions to manipulate and process data, generate charts and graphs, and interpret the results.

Time Frame of Unit: 3 – 4 weeks

Instructional Activities:

- CBL Labs (optional)
    - Knot Lab
    - Marble Roll
    - Weight/Spring Lab
    - Penny Lab
- Calculator Regression Analysis
- Calculator Investigation on the behavior of linear and absolute value equations.

Materials and Resources:

- McDougal Littell: Algebra 2
- TI-83 Graphing Calculators
- CBL

Unit of Study

Unit Title: Systems of Equations and Inequalities with algebraic methods and matrix operations.

Essential Questions of the Unit:

- How can a system of equations be used as a problem solving tool?
- How can matrices be used to organize real life data?

Assessments:

- Teacher made tests and quizzes
- CBL Data Collection Labs
- Project involving matrices
- Graphing Calculator Investigations

Content:

- 2 x 2 Linear Systems
- Linear Inequality Systems
- 3 x 3 Systems
- Matrix Operations – Addition, Subtraction and Multiplication
- Determinants
- Inverses of Matrices

Skills:

- To solve 2 x 2 systems of equations algebraically and graphically.
- To solve systems of linear inequalities graphically.
- To solve 3 x 3 systems of equations algebraically.
- To perform matrix operations with and without the graphing calculator.
- To find the determinant of a 2 x 2 and 3 x 3 matrix.
- To find the inverse of a 2 x 2 matrix.
- To solve systems of linear equations using matrices.
- To write systems of equations for word problems.
- To calculate the area of a triangle in the coordinate plane with matrices.

Purpose / Rationale of the Unit:

Systems of linear equations are used to model real life situations. The skills acquired in this section help students analyze and solve these situations.

New Jersey Core Curriculum Content Standards:

4.1 NUMBER AND NUMERICAL OPERATIONS

A. Number Sense -3
B. Numerical Operations – 1, 3

4.2 GEOMETRY AND MEASUREMENT

A. Geometric Properties - 2
B. Transforming Shapes - 1
C. Coordinate Geometry - 1

4.3 PATTERNS AND ALGEBRA

B. Functions – 1-3
D. Procedures – 2, 3

4.5 MATHEMATICAL PROCESSES

A. Problem Solving – 1-5

(For descriptive narrative, See Appendix)

Time Frame of Unit: 5 – 6 weeks

Instructional Activities:

- Traffic light project (matrices)
- Restaurant Nutritional Analysis (computer)
- CBL Intersection Lab
- Greenhouse Problem
- Linear Programming

Materials and Resources:

- McDougal Littell: Algebra 2
- TI-83 Graphing Calculators
- Online access for nutritional information on restaurants
- CBL

Unit of Study

Unit Title: Quadratic Equations

Essential Questions of the Unit:

- How can quadratic modeling be used in the analysis of real life data?
- When is it appropriate to use technology to enhance quadratic modeling?

Assessments:

- Teacher made tests and quizzes
- CBL Data Collection Labs
- Project involving analysis of data collected in a lab
- Graphing Calculator Investigations

Content:

- Trinomials
- Parabolas – Standard and Vertex Form
- Axis of Symmetry
- Quadratic Formula
- Imaginary and Complex Numbers
- Complex Number operations
- Difference of Two Squares

Skills:

- To factor trinomials in order to solve the quadratic.
- To graph parabolas with and without the calculator.
- To put a quadratic equation in standard form and in vertex form.
- To perform complex number operations.
- To solve quadratic equations for real and complex solutions.
- To look at real life situations and fit quadratic models to these situations.

Purpose / Rationale of the Unit:

Students should realize that most data is not linear and that many physical phenomena exhibit parabolic tendencies.

New Jersey Core Curriculum Content Standards:

4.2 GEOMETRY AND MEASUREMENT

A. Geometric Properties - 3
B. Transforming Shapes - 4

4.3 PATTERNS AND ALGEBRA

B. Functions - 1-4
C. Modeling - 1
D. Procedures – 1-3

4.4 DATA ANALYSIS, PROBABILITY, AND DISCRETE MATHEMATICS

A. Data Analysis (Statistics) - 4

4.5 MATHEMATICAL PROCESSES

A. Problem Solving – 1-5

(For descriptive narrative, See Appendix)

Time Frame of Unit: 6 – 8 weeks

Instructional Activities:

- CBL Motion Labs
- Graphing Calculator Activities on parabolic motion
- Video Point Analysis

Materials and Resources:

- McDougal Littell: Algebra 2
- TI-83 Graphing Calculators
- CBL
- VideoPoint software

Unit of Study

Unit Title: Functions

Essential Questions of the Unit:

- How are functions used to organize and manage mathematical concepts?
- How can composition be utilized efficiently to combine 2 or more mathematical formulas?
- How are transformations of parabolas and absolute value functions related to other functions?

Assessments:

- Teacher made tests and quizzes
- Green Globs Computer Software
- Project involving shifts of graphs
- Programming with recursive functions
- Graphing Calculator Investigations
- Computers with Video Point Software

Content:

- Relation
- Function
- Domain
- Range
- Function operations
- Inverses
- Compound functions
- Step functions reflections
- Translations
- Recursive functions

Skills:

- To identify and display relations and functions.
- To identify the domain and range of a function and relation..
- To find the composition of functions.
- To perform function operations.
- To find the inverses of functions.
- To verify 2 functions are inverses.
- To graph and use special functions.
- To use translations and reflections to sketch graphs.
- To use and write recursive functions.
- To use functions with problem solving.

Purpose / Rationale of the Unit:

Functions are used to represent real life situations and enable students to manipulate graphs of higher order polynomials. The concepts mastered in this chapter are essential in the further study of algebra.

New Jersey Core Curriculum Content Standards:

4.1 NUMBER AND NUMERICAL OPERATIONS
- B. Numerical Operations - 2
- C. Estimation - 1

4.2 GEOMETRY AND MEASUREMENT
- E. Measuring Geometric Objects - 2

4.3 PATTERNS AND ALGEBRA
- B. Functions – 1, 2, 4
- C. Modeling - 1
- D. Procedures – 1, 2, 3

4.5 MATHEMATICAL PROCESSES
- A. Problem Solving – 1-5

(For descriptive narrative, See Appendix)

Time Frame of Unit: 4 – 5 weeks

Instructional Activities:

- Writing calculator programs using recursive functions.
- Shifting any type of graph based on knowledge of parabolic and absolute value shifts.
- Investigating shifts with Green Globs.
- Solving word problems from Math Connections 1B.
- Stepping to the Greatest Integer CBL activity
- Quadratic CBL activities

Materials and Resources:

- McDougal Littell: Algebra 2
- TI-83 Graphing Calculators
- CBL
- Computers with Green Globs Software
- Sections 6.2, 6.3 in Math Connections 1B (Integrated 2 textbook)

Unit of Study

Unit Title: Powers, Roots and Radicals

Essential Questions of the Unit:

- How can an exponential function be used to model growth or decay?
- How are radicals and exponents used in problem solving?
- What's the relationship between a radical expression and an exponential expression?

Assessments:

- Teacher made tests and quizzes
- Green Globs Computer Software
- Project involving Compound Interest
- Real life Exponential Growth and Decay Data Sets
- Graphing Calculator Investigations
- CBL investigations

Content:

- Exponent properties
- Compound interest
- Exponential growth and decay
- Nth roots
- Root properties
- Radical properties
- Radical equations
- Distance formula
- Graphs of square and cube root functions

Skills:

- To use the properties of exponents and radicals.
- To calculate compound interest.
- To identify and write models of exponential growth and decay.
- To find the nth root of a number.
- To solve equations with radicals and roots.
- To sketch and transform graphs of square root and cube root functions.

Purpose / Rationale of the Unit:

To provide students with another way to model real life situations involving science and finances.

New Jersey Core Curriculum Content Standards:

4.1 NUMBER AND NUMERICAL OPERATIONS

A. Number Sense - 1
B. Numerical Operations -2

4.3 PATTERNS AND ALGEBRA

B. Functions – 2, 4
C. Modeling - 1
D. Procedures – 2, 3

4.4 DATA ANALYSIS, PROBABILITY, AND DISCRETE MATHEMATICS

A. Data Analysis (Statistics) – 2, 5

4.5 MATHEMATICAL PROCESSES

A. Problem Solving – 2, 4, 5

(For descriptive narrative, See Appendix)

Time Frame of Unit: 4 – 5 weeks

Instructional Activities:

- Compound Interest Project with banks – local and online
- Radioactive Decay activity (popcorn)
- Sour Chemistry CBL activity
- Too Hot to Trot CBL activity
- Bounce Back CBL activity
- Name that Tune activity
- M&M activity

Materials and Resources:

- McDougal Littell: Algebra 2
- TI-83 Graphing Calculators
- CBL
- Online access for bank project

Unit of Study

Unit Title: Rational Functions

Essential Questions of the Unit:

- How can situations in physics, economics and social studies be modeled using rational functions?
- What are practical applications to simplifying rational expressions?
- How do some characteristics of rational functions relate to the ideas of limits and infinity?

Assessments:

- Teacher made tests and quizzes
- Project involving Inverse Variation and Telescopes
- Graphing Calculator Investigations
- CBL investigations

Content:

- Cube of binomial
- Squares of binomials
- Factoring with grouping
- Asymptotes
- Rational expressions
- Complex fractions
- Inverse variation
- Direct variation
- Joint variation

Skills:

- To factor sum and difference of cubes.
- To factor using grouping
- To sketch graphs of rational functions.
- To identify vertical and horizontal asymptotes.
- To simplify rational expressions.
- To perform operations with rational expressions.
- To solve rational equations.
- To solve and writing models of inverse, direct and joint variation.

Purpose / Rationale of the Unit:

This unit introduces and explores concepts such as limits and infinity which will serve as a foundation for the students' study of calculus. The models of variation occur often in real life situations.

New Jersey Core Curriculum Content Standards:

4.3 PATTERNS AND ALGEBRA

B. Functions - 2
C. Modeling - 1
D. Procedures -1

4.5 MATHEMATICAL PROCESSES

A. Problem Solving – 1, 2, 3, 5

(For descriptive narrative, See Appendix)

Time Frame of Unit: 4 – 5 weeks

Instructional Activities:

- Under Pressure CBL activity
- "Light at a Distance" CBL activity
- Internet "Blue Book" Value Project
- Exploring Limits Activity
- Inverse and Direct Variation with Telescopes Lab

Materials and Resources:

- McDougal Littell: Algebra 2
- TI-83 Graphing Calculators
- CBL

Unit of Study

Unit Title: Conic Sections

Essential Questions of the Unit:

- What technological advances and simple devices utilize conics?
- How do you identify which conic an equation represents?
- How are conics modeled in scientific phenomena?

Assessments:

- Teacher made tests and quizzes
- Project involving Applications of Conics
- Graphing Calculator Investigations
- Student Directed Presentations on Conics

Content:

- Parabola
- Focus
- Directrix
- Circles
- Points of intersection
- Distance formula
- Ellipses
- Center
- Vertex
- Transverse, major and minor axes
- Asymptotes
- Translations

Skills:

- To graph conic sections.
- To identify the vertex, focus, axis of symmetry and directrix of a parabola.
- To identify the center and radius of a circle.
- To identify the center, major axis, minor axis and foci of an ellipse.
- To identify the vertices, foci and transverse axes of hyperbolas.
- To write equations of conics given properties.
- To identify real life applications of conics.
- To translate conics.
- To classify conics.

Purpose / Rationale of the Unit:

Students will see the many applications of conic sections in real life situations. This unit also explores concepts what will be further studied in calculus.

New Jersey Core Curriculum Content Standards:

4.2 GEOMETRY AND MEASUREMENT

A. Geometric Properties - 3
B. Transforming Shapes - 4
C. Coordinate Geometry - 1

4.3 PATTERNS AND ALGEBRA

B. Functions – 1, 4
D. Procedures - 2

4.5 MATHEMATICAL PROCESSES

A. Problem Solving - 5

(For descriptive narrative, See Appendix)

Time Frame of Unit: 3 – 4 weeks

Instructional Activities:

- Find or take pictures of conic in real life settings.
- Investigate applications of conics on the Internet.
- Introduce conics with student directed presentations on their basic characteristics. (jigsaw activity)

Materials and Resources:

- McDougal Littell: Algebra 2
- TI-83 Graphing Calculators
- Internet

New Jersey Core Curriculum Content Standards for

Mathematics

INTRODUCTION

The Vision

The vision of the mathematics standards is focused on achieving one crucial goal:

To enable ALL of New Jersey's children to acquire the mathematical skills, understandings, and attitudes that they will need to be successful in their careers and daily lives.

We want ALL students to achieve the standards. There may be exceptions, but those exceptions should be exceptional.

Perhaps the most compelling reason for this vision is that all of our children, as well as our state and our nation, will be better served by higher expectations, by curricula that go far beyond basic skills and include a variety of mathematical models, and by programs which devote a greater percentage of instructional time to problem-solving and active learning.

Many students respond to the traditional curriculum with boredom and discouragement. They feel that mathematics will never be useful in their lives, and they develop the perception that success in mathematics depends on some innate ability that they simply do not have.[1] We must overcome the feelings among students that they don't like mathematics, they don't need mathematics, and they can't do mathematics. Curricula that evoke these responses in students, curricula that assume student failure, are bound to fail; we need to develop curricula that assume student success.

Our curricula are often preoccupied with what national reports call "shopkeeper arithmetic,"[2] competency in the basic operations that were needed to run a small store several generations ago. The economy in which graduates of our schools will seek employment is more competitive than ever and is rapidly changing in response to advances in technology. To compete in today's global, information-based economy, students must be able to solve real problems, reason effectively, and make logical connections.

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[1] "Only in the United States do people believe that learning mathematics depends on special ability. In other countries, students, parents, and teachers all expect that most students can master mathematics if only they work hard enough. The record of accomplishment in these countries — and in some intervention programs in the United States — shows that most students can learn much more mathematics than is commonly assumed in this country." *Everybody Counts*, Mathematical Sciences Education Board, National Academy of Sciences (1989)

[2] *Everybody Counts*, Mathematical Sciences Education Board National Academy of Sciences (1989).

American schools have done well in the past at producing a relatively small mathematical elite that adequately served the needs of an industrial/mechanical economy. But that level of "production" is no longer good enough. Our state and our country need people with the skills to develop and manage these new technologies. Jobs increasingly require mathematical knowledge and skills in areas such as data analysis, problem-solving, pattern recognition, statistics, and probability. We must not only strive to provide our graduates with the skills for 21st century jobs, but also to ensure that the number of graduates with those skills is sufficient for the needs of our state and our nation.

This vision of excellent mathematical education is based on the twin premises that *all* students *can* learn mathematics and that all students *need* to learn mathematics. These mathematics standards were not designed as minimum standards, but rather as world-class standards which will enable all of our students to compete in the global marketplace of the 21st century.

The vision of success for all students in mathematics depends on:

- establishing learning environments that facilitate student learning of mathematics;
- a commitment to equity and to excellence; and
- defining the critical goals of mathematics education today--what students should know and be able to do (i.e., content and processes).

These three themes are discussed in the next three sections.

The mathematics standards are intended to be a definition of excellent practice, and a description of what can be achieved if all New Jersey communities rally behind the standards, so that this excellent practice becomes common practice. Making the vision a reality is an achievable goal.

The Vision – Learning Environments

The vision, if it is to be realized, must include learning environments with the following characteristics, as described in the mathematics standards adopted in 1996:

Students excited by and interested in their activities. A principal aim is for children to learn to enjoy mathematics. Students who are excited by what they are doing are more likely to truly understand the material, to stay involved over a longer period of time, and to take more advanced courses voluntarily. When math is taught with a problem-solving spirit, and when children are allowed to make their own hands-on mathematical discoveries, math can be engaging for all students.

Students learning important mathematical concepts rather than simply memorizing and practicing procedures. Student learning should be focused on understanding when and how mathematics is used and how to apply mathematical concepts. With the availability of technology, students need no longer spend the same amount of study time practicing lengthy computational processes. More effort should be devoted to the development of number sense, spatial sense, and estimation skills.

Students posing and solving meaningful problems. When students are challenged to use mathematics in meaningful ways, they develop their reasoning and problem-solving skills and come to realize the potential usefulness of mathematics in their lives.

Students working together to learn mathematics. Children learn mathematics well in cooperative settings, where they can share ideas and approaches with their classmates.

Students writing and talking about math topics every day. Putting thoughts into words helps to clarify and solidify thinking. By sharing their mathematical understandings in written and oral form with their classmates, teachers, and parents, students develop confidence in themselves as mathematical learners; this practice also enables teachers to better monitor student progress.

Students using calculators and computers as important tools of learning. Technology can be used to aid teaching and learning, as new concepts are presented through explorations with calculators or computers. But technology can also be used to assist students in solving problems, as it is used by adults in our society. Students should have access to these tools, both in school and after school, whenever they can use technology to do more powerful mathematics than they would otherwise be able to do.

Students whose teachers who have high expectations for ALL of their students. This vision includes a set of achievable, high-level expectations for the mathematical understanding and performance of all students. Although more ambitious than current expectations for most students, these standards are absolutely essential if we are to reach our goal. Those students who can achieve more than this set of expectations must be afforded the opportunity and encouraged to do so.

Students being assessed by a variety of assessment strategies, not just traditional short-answer tests. Strategies including open-ended problems, teacher interviews, portfolios of best work, and projects, in combination with traditional methods, will provide a more complete picture of students' performance and progress.

The Vision – Equity and Excellence

In order for all their students to succeed in mathematics, districts will need to commit themselves to the principles of equity and excellence, which comprised Standard 16 in the 1996 version of the mathematics standards, and which remain an important priority for all New Jersey schools. The equity and excellence component of the vision has four features:

Fostering respect for the power of mathematics. All students should learn that mathematics is integral to the development of all cultures and civilizations, and in particular to the advances in our own society. They should be aware that the adults in their world (parents, relatives, mentors, community members, role models) use mathematics on a daily basis. And they should know that success in mathematics may be a critical gateway to success in their careers, citizenship, and lives.

Setting high expectations. All students should have high expectations of themselves. These high expectations should be fostered by their teachers, administrators, and parents all of whom should themselves believe that all students can and will succeed in mathematics. This belief in his or her abilities often makes it possible for a child to succeed**.**

Providing opportunities for success. High expectations can only be achieved if students are provided with the appropriate opportunities. At all grade levels, students should receive instruction by teachers who have had the training and professional development appropriate for their grade level. Students should receive prompt and appropriate services essential to ensure that they can learn the mathematical skills and concepts included in the core curriculum, and to ensure that their weaknesses do not result in trapping them in a cycle of failure. Students should receive equitable treatment without regard to gender or ethnicity, and should not be conditioned to fail by predetermined low expectations.

Encouraging all students to go beyond the standards. Teachers should help students develop a positive attitude about mathematics by engaging them in exploring and solving interesting mathematical problems, by using mathematics in meaningful ways, by focusing on concepts and understanding as well as on rules and procedures, and by consistently expecting them to go beyond repetition and memorization to problem solving and understanding. Every effort should be made to ensure that all students are continuously encouraged, nurtured, and challenged to maximize their potential at all grade levels and to become prepared for college-level mathematics. Students who have achieved the standards should be encouraged to go beyond the standards. If schools challenge all students at lower grade levels, they will attain the goal of having advanced mathematics classrooms whose students reflect the diversity of the school's total population.

What Students Should Know and Be Able to Do

New Jersey's mathematics standards rest on the notion that an appropriate mathematics curriculum results from a series of critical decisions about three inseparably linked components: content, instruction, and assessment. The standards will only promote substantial and systemic improvement in mathematics education if the *what* of the content being learned, the *how* of the problem-solving orientation, and the *where* of the active, equitable, involving learning environment are synergistically woven together in every classroom. The mathematical environment of every child must be rich and complex and all students must be afforded the opportunity to develop an understanding and a command of mathematics in an environment that provides for both affective and intellectual growth.

Although ours is a geographically small state, it has a widely diverse population. Children enter our schools from a tremendous variety of backgrounds and cultures. One of the roles of New Jersey's mathematics standards, therefore, is to specify a set of achievable high-level expectations for the mathematical understanding and performance of *all* students. The expectations included in the standards are substantially more ambitious than traditional expectations for most students, but we believe that they are attainable by all students in the state. Those New Jersey students who can achieve more than this set of expectations must be afforded the opportunity and encouraged to do so.

Background

In May 1996, the New Jersey State Board of Education adopted Core Curriculum Content Standards, including a set of 16 standards in mathematics. The development and review of the 1996 version of the New Jersey's mathematics standards spanned a four-year period and involved two working panels and hundreds of educators and other citizens.

The adoption of the standards was followed in December by the publication of the *New Jersey Mathematics Curriculum Framework* that was developed to provide assistance and guidance to districts and teachers in how to implement these standards, in translating the vision into reality. The development of the framework was a joint effort of the New Jersey Mathematics Coalition and the New Jersey State Department of Education, with funding from the United States Department of Education.

New assessments have been introduced to reflect the new standards. The mathematics portions of New Jersey's Elementary School Proficiency Assessment (ESPA), Grade Eight Proficiency Assessment (GEPA), and the High School Proficiency Assessment (HSPA) are all based on the mathematics standards adopted by the State Board of Education.

The mathematics standards adopted in 1996 were philosophically aligned with the *Curriculum and Evaluation Standards for School Mathematics* of the National Council of Teachers of Mathematics (NCTM, 1989), but went beyond that document in a number of ways, reflecting national discussions of that document between 1989 and 1996 and taking into consideration conditions specific to New Jersey. Since 1996, NCTM has published a new document, *Principles and Standards for School Mathematics* (NCTM, 2000), and 49 of the 50 states have now adopted mathematics standards.

Revised Standards

The State Board of Education intended that a review of the standards take place after five years. The panel that drafted these revised standards, in preparing its recommendations, reviewed many of the state standards as well as *Principles and Standards for School Mathematics* (NCTM, 2000). The panel also took into consideration a review of New Jersey's 1996 standards prepared by Achieve, Inc. with the support of the Department of Education and Prudential. The panel kept in mind two important principles:

Retain the *content* of the current standards and the structure of the current assessments, so that the standards will not be a major departure from what is currently expected of students.

Revise the *presentation* of the standards, so that teachers will find them easier to understand and implement, and so that standards and assessments are better aligned.

The content of the new mathematics standards is therefore largely the same as the previous version. However, the new standards are different in that:

The new standards are more specific and clearer than the previous standards;

The new standards are organized into a smaller number of standards that correspond to the content clusters of the statewide assessments;

The new standards are intended to serve as clear guides to the assessment development committees so that there should be no gaps between the standards and the test specifications; and

The new standards include expectations at grades 2, 3, 5, 6, and 7, as well as at grades 4, 8, and 12.

Standards and Strands

There are five standards altogether, each of which has a number of lettered strands. These standards, and their associated strands, are enumerated below:

4.1. Number and Numerical Operations
A. Number Sense
B. Numerical Operations
C. Estimation

4.2. Geometry and Measurement
A. Geometric Properties
B. Transforming Shapes
C. Coordinate Geometry
D. Units of Measurement
E. Measuring Geometric Objects

4.3. Patterns and Algebra
A. Patterns and Relationships
B. Functions
C. Modeling
D. Procedures

4.4. Data Analysis, Probability, and Discrete Mathematics
A. Data Analysis (Statistics)
B. Probability
C. Discrete Mathematics--Systematic Listing and Counting
D. Discrete Mathematics--Vertex-Edge Graphs and Algorithms

4.5. Mathematical Processes
A. Problem Solving
B. Communication
C. Connections
D. Reasoning
E. Representations
F. Technology

The first four of these "standards" also serve as what have been called "content clusters" in the current state assessments; the lettered strands replace what have been called "macros" in the directories of test specifications. The fifth standard will continue to provide the "power base" of the assessments. It is anticipated that the expectations presented here will be used as the basis for test specifications for the next version of the statewide assessments.

For the first four standards, student expectations are provided for each strand at each of eight grade levels: 2, 3, 4, 5, 6, 7, 8, and 12. The expectations for the fifth standard are intended to address every grade level. With the exception of indicators for grades 3, 5, and 7, which were developed at a later time, items presented at one grade level are not generally repeated at subsequent grade levels. Teachers at each grade will need to refer to the standards at earlier grade levels to know what topics their students should have learned at earlier grades.

Bulleted items that appear below expectations indicate terminology, concepts, or content material addressed in that expectation. When an indicator is followed by bulleted content material, the list provided is intended to be exhaustive; content material not mentioned is therefore not included in the expectation at that grade level. When examples are provided, they are always introduced with "e.g." and are not intended to be exhaustive.

A Core Curriculum for Grades K-12

Implicit in the vision and standards is the notion that there should be a core curriculum for grades K-12. What does a "core curriculum" mean? It means that every student will be involved in experiences addressing all of the expectations of each of the content standards. It also means that all courses of study should have a common goal of completing this core curriculum, no matter how students are grouped or separated by needs and/or interests.

A core curriculum does not mean that all students will be enrolled in the same courses. Students have different aptitudes, interests, educational and professional plans, learning habits, and learning styles. Different groups of students should address the core curriculum at different levels of depth, and should complete the core curriculum according to different timetables. Nevertheless, all students should complete all elements of the core curriculum recommended in the mathematics standards.

All students should be challenged to reach their maximum potential. For many students, the core curriculum described here will indeed be challenging. But if we do not provide this challenge, we will be doing our students a great disservice — leaving them unprepared for the technological and information age of the 21st century.

For other students, this core curriculum itself will not be a challenge. We have to make sure that we provide these students with appropriate mathematical challenges. We have to make sure that the raised expectations for all students do not result in lowered expectations for our high achieving students. A core curriculum does not exclude a program that challenges students beyond the expectations set in the mathematics standards. Indeed, the vision of equity and excellence calls for schools to provide opportunities for their students to learn more mathematics than is contained in the core curriculum.

Summary

These refined mathematics standards, and the vision imbedded in them, offer a powerful challenge to all teachers, all schools, and all districts in New Jersey — to enable all of our students to step into this new century with the mathematical skills, understandings, and attitudes that they will need to be successful in their careers and daily lives. It will not be easy to meet this challenge, nor can it happen overnight. But it can happen if all of us together decide to make it happen. We must not let our awareness of the obstacles we face become yet another obstacle. We shall work together to make the vision of New Jersey's mathematics standards a reality!

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STANDARD 4.1 (NUMBER AND NUMERICAL OPERATIONS) ALL STUDENTS WILL DEVELOP NUMBER SENSE AND WILL PERFORM STANDARD NUMERICAL OPERATIONS AND ESTIMATIONS ON ALL TYPES OF NUMBERS IN A VARIETY OF WAYS.

Descriptive Statement: Numbers and arithmetic operations are what most of the general public think about when they think of mathematics; and, even though other areas like geometry, algebra, and data analysis have become increasingly important in recent years, numbers and operations remain at the heart of mathematical teaching and learning. Facility with numbers, the ability to choose the appropriate types of numbers and the appropriate operations for a given situation, and the ability to perform those operations as well as to estimate their results, are all skills that are essential for modern day life.

Number Sense. Number sense is an intuitive feel for numbers and a common sense approach to using them. It is a comfort with what numbers represent that comes from investigating their characteristics and using them in diverse situations. It involves an understanding of how different types of numbers, such as fractions and decimals, are related to each other, and how each can best be used to describe a particular situation. It subsumes the more traditional category of school mathematics curriculum called numeration and thus includes the important concepts of place value, number base, magnitude, and approximation and estimation.

Numerical Operations. Numerical operations are an essential part of the mathematics curriculum, especially in the elementary grades. Students must be able to select and apply various computational methods, including mental math, pencil-and-paper techniques, and the use of calculators. Students must understand how to add, subtract, multiply, and divide whole numbers, fractions, decimals, and other kinds of numbers. With the availability of calculators that perform these operations quickly and accurately, the instructional emphasis now is on understanding the meanings and uses of these operations, and on estimation and mental skills, rather than solely on the development of paper-and-pencil proficiency.

Estimation. Estimation is a process that is used constantly by mathematically capable adults, and one that can be easily mastered by children. It involves an educated guess about a quantity or an intelligent prediction of the outcome of a computation. The growing use of calculators makes it more important than ever that students know when a computed answer is reasonable; the best way to make that determination is through the use of strong estimation skills. Equally important is an awareness of the many situations in which an approximate answer is as good as, or even preferable to, an exact one. Students can learn to make these judgments and use mathematics more powerfully as a result.

Number and operation skills continue to be a critical piece of the school mathematics curriculum and, indeed, a very important part of mathematics. But, there is perhaps a greater need for us to rethink our approach here than to do so for any other curriculum component. An enlightened mathematics program for today's children will empower them to use all of today's tools rather than require them to meet yesterday's expectations.

Strands and Cumulative Progress Indicators

Building upon knowledge and skills gained in preceding grades, by the end of Grade 6, students will:

A. Number Sense

1. Use real-life experiences, physical materials, and technology to construct meanings for numbers (unless otherwise noted, all indicators for grade 6 pertain to these sets of numbers as well).
    - All integers
    - All fractions as part of a whole, as subset of a set, as a location on a number line, and as divisions of whole numbers
    - All decimals
2. Recognize the decimal nature of United States currency and compute with money.
3. Demonstrate a sense of the relative magnitudes of numbers.
4. Explore the use of ratios and proportions in a variety of situations.
5. Understand and use whole-number percents between 1 and 100 in a variety of situations.
6. Use whole numbers, fractions, and decimals to represent equivalent forms of the same number.
7. Develop and apply number theory concepts in problem solving situations.
    - Primes, factors, multiples
    - Common multiples, common factors
8. Compare and order numbers.

B. Numerical Operations

1. Recognize the appropriate use of each arithmetic operation in problem situations.
2. Construct, use, and explain procedures for performing calculations with fractions and decimals with:
    - Pencil-and-paper
    - Mental math
    - Calculator
3. Use an efficient and accurate pencil-and-paper procedure for division of a 3-digit number by a 2-digit number.
4. Select pencil-and-paper, mental math, or a calculator as the appropriate computational method in a given situation depending on the context and numbers.
5. Find squares and cubes of whole numbers.
6. Check the reasonableness of results of computations.
7. Understand and use the various relationships among operations and properties of operations.
8. Understand and apply the standard algebraic order of operations for the four basic operations, including appropriate use of parentheses.

C. Estimation

1. Use a variety of strategies for estimating both quantities and the results of computations.
2. Recognize when an estimate is appropriate, and understand the usefulness of an estimate as distinct from an exact answer.
3. Determine the reasonableness of an answer by estimating the result of operations.
4. Determine whether a given estimate is an overestimate or an underestimate.

Building upon knowledge and skills gained in preceding grades, by the end of Grade 7, students will:

A. Number Sense

1. Extend understanding of the number system by constructing meanings for the following (unless otherwise noted, all indicators for grade 7 pertain to these sets of numbers as well):
    - Rational numbers
    - Percents
    - Whole numbers with exponents
2. Demonstrate a sense of the relative magnitudes of numbers.
3. Understand and use ratios, proportions, and percents (including percents greater than 100 and less than 1) in a variety of situations.
4. Compare and order numbers of all named types.
5. Use whole numbers, fractions, decimals, and percents to represent equivalent forms of the same number.
6. Understand that all fractions can be represented as repeating or terminating decimals.

B. Numerical Operations

1. Use and explain procedures for performing calculations with integers and all number types named above with:
    - Pencil-and-paper
    - Mental math
    - Calculator
2. Use exponentiation to find whole number powers of numbers.
3. Understand and apply the standard algebraic order of operations, including appropriate use of parentheses.

C. Estimation

1. Use equivalent representations of numbers such as fractions, decimals, and percents to facilitate estimation.

Building upon knowledge and skills gained in preceding grades, by the end of Grade 8, students will:

A. Number Sense

1. Extend understanding of the number system by constructing meanings for the following (unless otherwise noted, all indicators for grade 8 pertain to these sets of numbers as well):
    - Rational numbers
    - Percents
    - Exponents
    - Roots
    - Absolute values
    - Numbers represented in scientific notation
2. Demonstrate a sense of the relative magnitudes of numbers.

3. Understand and use ratios, proportions, and percents (including percents greater than 100 and less than 1) in a variety of situations.
4. Compare and order numbers of all named types.
5. Use whole numbers, fractions, decimals, and percents to represent equivalent forms of the same number.
6. Recognize that repeating decimals correspond to fractions and determine their fractional equivalents.
    - 5/7 = 0. 714285714285… = $0.\overline{714285}$
7. Construct meanings for common irrational numbers, such as π (pi) and the square root of 2.

B. Numerical Operations

1. Use and explain procedures for performing calculations involving addition, subtraction, multiplication, division, and exponentiation with integers and all number types named above with:
    - Pencil-and-paper
    - Mental math
    - Calculator
2. Use exponentiation to find whole number powers of numbers.
3. Find square and cube roots of numbers and understand the inverse nature of powers and roots.
4. Solve problems involving proportions and percents.
5. Understand and apply the standard algebraic order of operations, including appropriate use of parentheses.

C. Estimation

1. Estimate square and cube roots of numbers.
2. Use equivalent representations of numbers such as fractions, decimals, and percents to facilitate estimation.
3. Recognize the limitations of estimation and assess the amount of error resulting from estimation.

Building upon knowledge and skills gained in preceding grades, by the end of Grade 12, students will:

A. Number Sense

1. Extend understanding of the number system to all real numbers.
2. Compare and order rational and irrational numbers.
3. Develop conjectures and informal proofs of properties of number systems and sets of numbers.

B. Numerical Operations

1. Extend understanding and use of operations to real numbers and algebraic procedures.
2. Develop, apply, and explain methods for solving problems involving rational and negative exponents.

3. Perform operations on matrices.
    - Addition and subtraction
    - Scalar multiplication

4. Understand and apply the laws of exponents to simplify expressions involving numbers raised to powers.

C. Estimation

1. Recognize the limitations of estimation, assess the amount of error resulting from estimation, and determine whether the error is within acceptable tolerance limits.

STANDARD 4.2 (GEOMETRY AND MEASUREMENT) ALL STUDENTS WILL DEVELOP SPATIAL SENSE AND THE ABILITY TO USE GEOMETRIC PROPERTIES, RELATIONSHIPS, AND MEASUREMENT TO MODEL, DESCRIBE AND ANALYZE PHENOMENA.

Descriptive Statement: Spatial sense is an intuitive feel for shape and space. Geometry and measurement both involve describing the shapes we see all around us in art, nature, and the things we make. Spatial sense, geometric modeling, and measurement can help us to describe and interpret our physical environment and to solve problems.

Geometric Properties. This includes identifying, describing and classifying standard geometric objects, describing and comparing properties of geometric objects, making conjectures concerning them, and using reasoning and proof to verify or refute conjectures and theorems. Also included here are such concepts as symmetry, congruence, and similarity.

Transforming Shapes. Analyzing how various transformations affect geometric objects allows students to enhance their spatial sense. This includes combining shapes to form new ones and decomposing complex shapes into simpler ones. It includes the standard geometric transformations of translation (slide), reflection (flip), rotation (turn), and dilation (scaling). It also includes using tessellations and fractals to create geometric patterns.

Coordinate Geometry. Coordinate geometry provides an important connection between geometry and algebra. It facilitates the visualization of algebraic relationships, as well as an analytical understanding of geometry.

Units of Measurement. Measurement helps describe our world using numbers. An understanding of how we attach numbers to real-world phenomena, familiarity with common measurement units (e.g., inches, liters, and miles per hour), and a practical knowledge of measurement tools and techniques are critical for students' understanding of the world around them.

Measuring Geometric Objects. This area focuses on applying the knowledge and understandings of units of measurement in order to actually perform measurement. While students will eventually apply formulas, it is important that they develop and apply strategies that derive from their understanding of the attributes. In addition to measuring objects directly, students apply indirect measurement skills, using, for example, similar triangles and trigonometry.

Students of all ages should realize that geometry and measurement are all around them. Through study of these areas and their applications, they should come to better understand and appreciate the role of mathematics in their lives.

Strands and Cumulative Progress Indicators

Building upon knowledge and skills gained in preceding grades, by the end of Grade 6, students will:

A. Geometric Properties

1. Understand and apply concepts involving lines and angles.
    - Notation for line, ray, angle, line segment
    - Properties of parallel, perpendicular, and intersecting lines
    - Sum of the measures of the interior angles of a triangle is 180°
2. Identify, describe, compare, and classify polygons and circles.
    - Triangles by angles and sides
    - Quadrilaterals, including squares, rectangles, parallelograms, trapezoids, rhombi
    - Polygons by number of sides.
    - Equilateral, equiangular, regular
    - All points equidistant from a given point form a circle
3. Identify similar figures.
4. Understand and apply the concepts of congruence and symmetry (line and rotational).
5. Compare properties of cylinders, prisms, cones, pyramids, and spheres.
6. Identify, describe, and draw the faces or shadows (projections) of three-dimensional geometric objects from different perspectives.
7. Identify a three-dimensional shape with given projections (top, front and side views).
8. Identify a three-dimensional shape with a given net (i.e., a flat pattern that folds into a 3D shape).

B. Transforming Shapes

1. Use a translation, a reflection, or a rotation to map one figure onto another congruent figure.
2. Recognize, identify, and describe geometric relationships and properties as they exist in nature, art, and other real-world settings.

C. Coordinate Geometry

1. Create geometric shapes with specified properties in the first quadrant on a coordinate grid.

D. Units of Measurement

1. Select and use appropriate units to measure angles, area, surface area, and volume.
2. Use a scale to find a distance on a map or a length on a scale drawing.
3. Convert measurement units within a system (e.g., 3 feet = ___ inches).
4. Know approximate equivalents between the standard and metric systems (e.g., one kilometer is approximately 6/10 of a mile).
5. Use measurements and estimates to describe and compare phenomena.

E. Measuring Geometric Objects

1. Use a protractor to measure angles.
2. Develop and apply strategies and formulas for finding perimeter and area.
    - Triangle, square, rectangle, parallelogram, and trapezoid
    - Circumference and area of a circle
3. Develop and apply strategies and formulas for finding the surface area and volume of rectangular prisms and cylinders.
4. Recognize that shapes with the same perimeter do not necessarily have the same area and vice versa.
5. Develop informal ways of approximating the measures of familiar objects (e.g., use a grid to approximate the area of the bottom of one's foot).

Building upon knowledge and skills gained in preceding grades, by the end of Grade 7, students will:

A. Geometric Properties

1. Understand and apply properties of polygons.
    - Quadrilaterals, including squares, rectangles, parallelograms, trapezoids, rhombi
    - Regular polygons
2. Understand and apply the concept of similarity.
    - Using proportions to find missing measures
    - Scale drawings
    - Models of 3D objects
3. Use logic and reasoning to make and support conjectures about geometric objects.

B. Transforming Shapes

1. Understand and apply transformations.
    - Finding the image, given the pre-image, and vice-versa
    - Sequence of transformations needed to map one figure onto another
    - Reflections, rotations, and translations result in images congruent to the pre-image
    - Dilations (stretching/shrinking) result in images similar to the pre-image

C. Coordinate Geometry

1. Use coordinates in four quadrants to represent geometric concepts.
2. Use a coordinate grid to model and quantify transformations (e.g., translate right 4 units).

D. Units of Measurement

1. Solve problems requiring calculations that involve different units of measurement within a measurement system (e.g., 4'3" plus 7'10" equals 12'1").
2. Select and use appropriate units and tools to measure quantities to the degree of precision needed in a particular problem-solving situation.
3. Recognize that all measurements of continuous quantities are approximations.

E. Measuring Geometric Objects

1. Develop and apply strategies for finding perimeter and area.
    - Geometric figures made by combining triangles, rectangles and circles or parts of circles
    - Estimation of area using grids of various sizes
2. Recognize that the volume of a pyramid or cone is one-third of the volume of the prism or cylinder with the same base and height (e.g., use rice to compare volumes of figures with same base and height).

Building upon knowledge and skills gained in preceding grades, by the end of Grade 8, students will:

A. Geometric Properties

1. Understand and apply concepts involving lines, angles, and planes.
    - Complementary and supplementary angles
    - Vertical angles
    - Bisectors and perpendicular bisectors
    - Parallel, perpendicular, and intersecting planes
    - Intersection of plane with cube, cylinder, cone, and sphere
2. Understand and apply the Pythagorean theorem.
3. Understand and apply properties of polygons.
    - Quadrilaterals, including squares, rectangles, parallelograms, trapezoids, rhombi
    - Regular polygons
    - Sum of measures of interior angles of a polygon
    - Which polygons can be used alone to generate a tessellation and why
4. Understand and apply the concept of similarity.
    - Using proportions to find missing measures
    - Scale drawings
    - Models of 3D objects
5. Use logic and reasoning to make and support conjectures about geometric objects.

B. Transforming Shapes

1. Understand and apply transformations.
    - Finding the image, given the pre-image, and vice-versa
    - Sequence of transformations needed to map one figure onto another
    - Reflections, rotations, and translations result in images congruent to the pre-image
    - Dilations (stretching/shrinking) result in images similar to the pre-image
2. Use iterative procedures to generate geometric patterns.
    - Fractals (e.g., the Koch Snowflake)
    - Self-similarity
    - Construction of initial stages
    - Patterns in successive stages (e.g., number of triangles in each stage of Sierpinski's Triangle)

C. Coordinate Geometry

1. Use coordinates in four quadrants to represent geometric concepts.
2. Use a coordinate grid to model and quantify transformations (e.g., translate right 4 units).

D. Units of Measurement

1. Solve problems requiring calculations that involve different units of measurement within a measurement system (e.g., 4’3" plus 7’10" equals 12’1").
2. Use approximate equivalents between standard and metric systems to estimate measurements (e.g., 5 kilometers is about 3 miles).
3. Recognize that the degree of precision needed in calculations depends on how the results will be used and the instruments used to generate the measurements.
4. Select and use appropriate units and tools to measure quantities to the degree of precision needed in a particular problem-solving situation.
5. Recognize that all measurements of continuous quantities are approximations.
6. Solve problems that involve compound measurement units, such as speed (miles per hour), air pressure (pounds per square inch), and population density (persons per square mile).

E. Measuring Geometric Objects

1. Develop and apply strategies for finding perimeter and area.
    - Geometric figures made by combining triangles, rectangles and circles or parts of circles
    - Estimation of area using grids of various sizes
    - Impact of a dilation on the perimeter and area of a 2-dimensional figure
2. Recognize that the volume of a pyramid or cone is one-third of the volume of the prism or cylinder with the same base and height (e.g., use rice to compare volumes of figures with same base and height).
3. Develop and apply strategies and formulas for finding the surface area and volume of a three-dimensional figure.
    - Volume - prism, cone, pyramid
    - Surface area - prism (triangular or rectangular base), pyramid (triangular or rectangular base)
    - Impact of a dilation on the surface area and volume of a three-dimensional figure
4. Use formulas to find the volume and surface area of a sphere.

Building upon knowledge and skills gained in preceding grades, by the end of Grade 12, students will:

A. Geometric Properties

1. Use geometric models to represent real-world situations and objects and to solve problems using those models (e.g., use Pythagorean Theorem to decide whether an object can fit through a doorway).
2. Draw perspective views of 3D objects on isometric dot paper, given 2D representations (e.g., nets or projective views).
3. Apply the properties of geometric shapes.
    - Parallel lines – transversal, alternate interior angles, corresponding angles
    - Triangles

    a. Conditions for congruence

    b. Segment joining midpoints of two sides is parallel to and half the length of the third side

c. Triangle Inequality
- Minimal conditions for a shape to be a special quadrilateral
- Circles – arcs, central and inscribed angles, chords, tangents
- Self-similarity

4. Use reasoning and some form of proof to verify or refute conjectures and theorems.
- Verification or refutation of proposed proofs
- Simple proofs involving congruent triangles
- Counterexamples to incorrect conjectures

B. Transforming Shapes

1. Determine, describe, and draw the effect of a transformation, or a sequence of transformations, on a geometric or algebraic object, and, conversely, determine whether and how one object can be transformed to another by a transformation or a sequence of transformations.
2. Recognize three-dimensional figures obtained through transformations of two-dimensional figures (e.g., cone as rotating an isosceles triangle about an altitude), using software as an aid to visualization.
3. Determine whether two or more given shapes can be used to generate a tessellation.
4. Generate and analyze iterative geometric patterns.
- Fractals (e.g., Sierpinski's Triangle)
- Patterns in areas and perimeters of self-similar figures
- Outcome of extending iterative process indefinitely

C. Coordinate Geometry

1. Use coordinate geometry to represent and verify properties of lines.
- Distance between two points
- Midpoint and slope of a line segment
- Finding the intersection of two lines
- Lines with the same slope are parallel
- Lines that are perpendicular have slopes whose product is –1

2. Show position and represent motion in the coordinate plane using vectors.
- Addition and subtraction of vectors

D. Units of Measurement

1. Understand and use the concept of significant digits.
2. Choose appropriate tools and techniques to achieve the specified degree of precision and error needed in a situation.
- Degree of accuracy of a given measurement tool
- Finding the interval in which a computed measure (e.g., area or volume) lies, given the degree of precision of linear measurements

E. Measuring Geometric Objects

1. Use techniques of indirect measurement to represent and solve problems.
    - Similar triangles
    - Pythagorean theorem
    - Right triangle trigonometry (sine, cosine, tangent)
2. Use a variety of strategies to determine perimeter and area of plane figures and surface area and volume of 3D figures.
    - Approximation of area using grids of different sizes
    - Finding which shape has minimal (or maximal) area, perimeter, volume, or surface area under given conditions using graphing calculators, dynamic geometric software, and/or spreadsheets
    - Estimation of area, perimeter, volume, and surface area

STANDARD 4.3 (PATTERNS AND ALGEBRA) ALL STUDENTS WILL REPRESENT AND ANALYZE RELATIONSHIPS AMONG VARIABLE QUANTITIES AND SOLVE PROBLEMS INVOLVING PATTERNS, FUNCTIONS, AND ALGEBRAIC CONCEPTS AND PROCESSES.

Descriptive Statement: Algebra is a symbolic language used to express mathematical relationships. Students need to understand how quantities are related to one another, and how algebra can be used to concisely express and analyze those relationships. Modern technology provides tools for supplementing the traditional focus on algebraic procedures, such as solving equations, with a more visual perspective, with graphs of equations displayed on a screen. Students can then focus on understanding the relationship between the equation and the graph, and on what the graph represents in a real-life situation.

Patterns. Algebra provides the language through which we communicate the patterns in mathematics. From the earliest age, students should be encouraged to investigate the patterns that they find in numbers, shapes, and expressions, and, by doing so, to make mathematical discoveries. They should have opportunities to analyze, extend, and create a variety of patterns and to use pattern-based thinking to understand and represent mathematical and other real-world phenomena.

Functions and Relationships. The function concept is one of the most fundamental unifying ideas of modern mathematics. Students begin their study of functions in the primary grades, as they observe and study patterns. As students grow and their ability to abstract matures, students form rules, display information in a table or chart, and write equations which express the relationships they have observed. In high school, they use the more formal language of algebra to describe these relationships.

Modeling. Algebra is used to model real situations and answer questions about them. This use of algebra requires the ability to represent data in tables, pictures, graphs, equations or inequalities, and rules. Modeling ranges from writing simple number sentences to help solve story problems in the primary grades to using functions to describe the relationship between two variables, such as the height of a pitched ball over time. Modeling also includes some of the conceptual building blocks of calculus, such as how quantities change over time and what happens in the long run (limits).

Procedures. Techniques for manipulating algebraic expressions – procedures – remain important, especially for students who may continue their study of mathematics in a calculus program. Utilization of algebraic procedures includes understanding and applying properties of numbers and operations, using symbols and variables appropriately, working with expressions, equations, and inequalities, and solving equations and inequalities.

Algebra is a gatekeeper for the future study of mathematics, science, the social sciences, business, and a host of other areas. In the past, algebra has served as a filter, screening people out of these opportunities. For New Jersey to be part of the global society, it is important that algebra play a major role in a mathematics program that opens the gates for all students.

Strands and Cumulative Progress Indicators

Building upon knowledge and skills gained in preceding grades, by the end of Grade 6, students will:

A. Patterns

1. Recognize, describe, extend, and create patterns involving whole numbers and rational numbers.
    - Descriptions using tables, verbal rules, simple equations, and graphs
    - Formal iterative formulas (e.g., NEXT = NOW * 3)
    - Recursive patterns, including Pascal's Triangle (where each entry is the sum of the entries above it) and the Fibonacci Sequence: 1, 1, 2, 3, 5, 8, . . . (where NEXT = NOW + PREVIOUS)

B. Functions and Relationships

1. Describe the general behavior of functions given by formulas or verbal rules (e.g., graph to determine whether increasing or decreasing, linear or not).

C. Modeling

1. Use patterns, relations, and linear functions to model situations.
    - Using variables to represent unknown quantities
    - Using concrete materials, tables, graphs, verbal rules, algebraic expressions/equations/inequalities
2. Draw freehand sketches of graphs that model real phenomena and use such graphs to predict and interpret events.
    - Changes over time
    - Relations between quantities
    - Rates of change (e.g., when is plant growing slowly/rapidly, when is temperature dropping most rapidly/slowly)

D. Procedures

1. Solve simple linear equations with manipulatives and informally.
    - Whole-number coefficients only, answers also whole numbers
    - Variables on one or both sides of equation
2. Understand and apply the properties of operations and numbers.
    - Distributive property
    - The product of a number and its reciprocal is 1
3. Evaluate numerical expressions.
4. Extend understanding and use of inequality.
    - Symbols ($\geq$, $\neq$, $\leq$)

Building upon knowledge and skills gained in preceding grades, by the end of Grade 7, students will:

A. Patterns

1. Recognize, describe, extend, and create patterns involving whole numbers, rational numbers, and integers.
    - Descriptions using tables, verbal and symbolic rules, graphs, simple equations or expressions
    - Finite and infinite sequences
    - Generating sequences by using calculators to repeatedly apply a formula

B. Functions and Relationships

1. Graph functions, and understand and describe their general behavior.
    - Equations involving two variables

C. Modeling

1. Analyze functional relationships to explain how a change in one quantity can result in a change in another, using pictures, graphs, charts, and equations.
2. Use patterns, relations, symbolic algebra, and linear functions to model situations.
    - Using manipulatives, tables, graphs, verbal rules, algebraic expressions/equations/inequalities
    - Growth situations, such as population growth and compound interest, using recursive (e.g., NOW-NEXT) formulas (cf. science standard 5.5 and social studies standard 6.6)

D. Procedures

1. Use graphing techniques on a number line.
    - Absolute value
    - Arithmetic operations represented by vectors (arrows) (e.g., "-3 + 6" is "left 3, right 6")
2. Solve simple linear equations informally and graphically.
    - Multi-step, integer coefficients only (although answers may not be integers)
    - Using paper-and-pencil, calculators, graphing calculators, spreadsheets, and other technology
3. Create, evaluate, and simplify algebraic expressions involving variables.
    - Order of operations, including appropriate use of parentheses
    - Substitution of a number for a variable
4. Understand and apply the properties of operations, numbers, equations, and inequalities.
    - Additive inverse
    - Multiplicative inverse

Building upon knowledge and skills gained in preceding grades, by the end of Grade 8, students will:

A. Patterns

1. Recognize, describe, extend, and create patterns involving whole numbers, rational numbers, and integers.
    - Descriptions using tables, verbal and symbolic rules, graphs, simple equations or expressions
    - Finite and infinite sequences
    - Arithmetic sequences (i.e., sequences generated by repeated addition of a fixed number, positive or negative)
    - Geometric sequences (i.e., sequences generated by repeated multiplication by a fixed positive ratio, greater than 1 or less than 1)
    - Generating sequences by using calculators to repeatedly apply a formula

B. Functions and Relationships

1. Graph functions, and understand and describe their general behavior.
    - Equations involving two variables
    - Rates of change (informal notion of slope)
2. Recognize and describe the difference between linear and exponential growth, using tables, graphs, and equations.

C. Modeling

1. Analyze functional relationships to explain how a change in one quantity can result in a change in another, using pictures, graphs, charts, and equations.
2. Use patterns, relations, symbolic algebra, and linear functions to model situations.
    - Using concrete materials (manipulatives), tables, graphs, verbal rules, algebraic expressions/equations/inequalities
    - Growth situations, such as population growth and compound interest, using recursive (e.g., NOW-NEXT) formulas (cf. science standard 5.5 and social studies standard 6.6)

D. Procedures

1. Use graphing techniques on a number line.
    - Absolute value
    - Arithmetic operations represented by vectors (arrows) (e.g., "-3 + 6" is "left 3, right 6")
2. Solve simple linear equations informally, graphically, and using formal algebraic methods.
    - Multi-step, integer coefficients only (although answers may not be integers)
    - Using paper-and-pencil, calculators, graphing calculators, spreadsheets, and other technology
3. Solve simple linear inequalities.
4. Create, evaluate, and simplify algebraic expressions involving variables.
    - Order of operations, including appropriate use of parentheses
    - Distributive property
    - Substitution of a number for a variable
    - Translation of a verbal phrase or sentence into an algebraic expression, equation, or inequality, and vice versa

5. Understand and apply the properties of operations, numbers, equations, and inequalities.
    - Additive inverse
    - Multiplicative inverse
    - Addition and multiplication properties of equality
    - Addition and multiplication properties of inequalities

Building upon knowledge and skills gained in preceding grades, by the end of Grade 12, students will:

A. Patterns

1. Use models and algebraic formulas to represent and analyze sequences and series.
    - Explicit formulas for n^{th} terms
    - Sums of finite arithmetic series
    - Sums of finite and infinite geometric series
2. Develop an informal notion of limit.
3. Use inductive reasoning to form generalizations.

B. Functions and Relationships

1. Understand relations and functions and select, convert flexibly among, and use various representations for them, including equations or inequalities, tables, and graphs.
2. Analyze and explain the general properties and behavior of functions of one variable, using appropriate graphing technologies.
    - Slope of a line or curve
    - Domain and range
    - Intercepts
    - Continuity
    - Maximum/minimum
    - Estimating roots of equations
    - Intersecting points as solutions of systems of equations
    - Rates of change
3. Understand and perform transformations on commonly-used functions.
    - Translations, reflections, dilations
    - Effects on linear and quadratic graphs of parameter changes in equations
    - Using graphing calculators or computers for more complex functions
4. Understand and compare the properties of classes of functions, including exponential, polynomial, rational, and trigonometric functions.
    - Linear vs. non-linear
    - Symmetry
    - Increasing/decreasing on an interval

C. Modeling

1. Use functions to model real-world phenomena and solve problems that involve varying quantities.
    - Linear, quadratic, exponential, periodic (sine and cosine), and step functions (e.g., price of mailing a first-class letter over the past 200 years)
    - Direct and inverse variation
    - Absolute value
    - Expressions, equations and inequalities
    - Same function can model variety of phenomena
    - Growth/decay and change in the natural world
    - Applications in mathematics, biology, and economics (including compound interest)

2. Analyze and describe how a change in an independent variable leads to change in a dependent one.
3. Convert recursive formulas to linear or exponential functions (e.g., Tower of Hanoi and doubling).

D. Procedures

1. Evaluate and simplify expressions.
    - Add and subtract polynomials
    - Multiply a polynomial by a monomial or binomial
    - Divide a polynomial by a monomial

2. Select and use appropriate methods to solve equations and inequalities.
    - Linear equations – algebraically
    - Quadratic equations – factoring (when the coefficient of x^2 is 1) and using the quadratic formula
    - All types of equations using graphing, computer, and graphing calculator techniques
3. Judge the meaning, utility, and reasonableness of the results of symbol manipulations, including those carried out by technology.

STANDARD 4.4 (DATA ANALYSIS, PROBABILITY, AND DISCRETE MATHEMATICS) ALL STUDENTS WILL DEVELOP AN UNDERSTANDING OF THE CONCEPTS AND TECHNIQUES OF DATA ANALYSIS, PROBABILITY, AND DISCRETE MATHEMATICS, AND WILL USE THEM TO MODEL SITUATIONS, SOLVE PROBLEMS, AND ANALYZE AND DRAW APPROPRIATE INFERENCES FROM DATA.

Descriptive Statement: Data analysis, probability, and discrete mathematics are important interrelated areas of applied mathematics. Each provides students with powerful mathematical perspectives on everyday phenomena and with important examples of how mathematics is used in the modern world. Two important areas of discrete mathematics are addressed in this standard; a third area, iteration and recursion, is addressed in Standard 4.3 (Patterns and Algebra).

Data Analysis (or Statistics). In today's information-based world, students need to be able to read, understand, and interpret data in order to make informed decisions. In the early grades, students should be involved in collecting and organizing data, and in presenting it using tables, charts, and graphs. As they progress, they should gather data using sampling, and should increasingly be expected to analyze and make inferences from data, as well as to analyze data and inferences made by others.

Probability. Students need to understand the fundamental concepts of probability so that they can interpret weather forecasts, avoid unfair games of chance, and make informed decisions about medical treatments whose success rate is provided in terms of percentages. They should regularly be engaged in predicting and determining probabilities, often based on experiments (like flipping a coin 100 times), but eventually based on theoretical discussions of probability that make use of systematic counting strategies. High school students should use probability models and solve problems involving compound events and sampling.

Discrete Mathematics—Systematic Listing and Counting. Development of strategies for listing and counting can progress through all grade levels, with middle and high school students using the strategies to solve problems in probability. Primary students, for example, might find all outfits that can be worn using two coats and three hats; middle school students might systematically list and count the number of routes from one site on a map to another; and high school students might determine the number of three-person delegations that can be selected from their class to visit the mayor.

Discrete Mathematics—Vertex-Edge Graphs and Algorithms. Vertex-edge graphs, consisting of dots (vertices) and lines joining them (edges), can be used to represent and solve problems based on real-world situations. Students should learn to follow and devise lists of instructions, called "algorithms," and use algorithmic thinking to find the best solution to problems like those involving vertex-edge graphs, but also to solve other problems.

These topics provide students with insight into how mathematics is used by decision-makers in our society, and with important tools for modeling a variety of real-world situations. Students will better understand and interpret the vast amounts of quantitative data that they are exposed to daily, and they will be able to judge the validity of data-supported arguments.

Strands and Cumulative Progress Indicators

Building upon knowledge and skills gained in preceding grades, by the end of Grade 6, students will:

A. Data Analysis

1. Collect, generate, organize, and display data.
    - Data generated from surveys

2. Read, interpret, select, construct, analyze, generate questions about, and draw inferences from displays of data.
    - Bar graph, line graph, circle graph, table, histogram
    - Range, median, and mean
    - Calculators and computers used to record and process information
3. Respond to questions about data, generate their own questions and hypotheses, and formulate strategies for answering their questions and testing their hypotheses.

B. Probability

1. Determine probabilities of events.
    - Event, complementary event, probability of an event
    - Multiplication rule for probabilities
    - Probability of certain event is 1 and of impossible event is 0
    - Probabilities of event and complementary event add up to 1
2. Determine probability using intuitive, experimental, and theoretical methods (e.g., using model of picking items of different colors from a bag).
    - Given numbers of various types of items in a bag, what is the probability that an item of one type will be picked
    - Given data obtained experimentally, what is the likely distribution of items in the bag
3. Explore compound events.
4. Model situations involving probability using simulations (with spinners, dice) and theoretical models.
5. Recognize and understand the connections among the concepts of independent outcomes, picking at random, and fairness.

C. Discrete Mathematics—Systematic Listing and Counting

1. Solve counting problems and justify that all possibilities have been enumerated without duplication.
    - Organized lists, charts, tree diagrams, tables
    - Venn diagrams
2. Apply the multiplication principle of counting.
    - Simple situations (e.g., you can make $3 \times 4 = 12$ outfits using 3 shirts and 4 skirts).
    - Number of ways a specified number of items can be arranged in order (concept of permutation)
    - Number of ways of selecting a slate of officers from a class (e.g., if there are 23 students and 3 officers, the number is $23 \times 22 \times 21$)

3. List the possible combinations of two elements chosen from a given set (e.g., forming a committee of two from a group of 12 students, finding how many handshakes there will be among ten people if everyone shakes each other person's hand once).

D. Discrete Mathematics—Vertex-Edge Graphs and Algorithms

1. Devise strategies for winning simple games (e.g., start with two piles of objects, each of two players in turn removes any number of objects from a single pile, and the person to take the last group of objects wins) and express those strategies as sets of directions.
2. Analyze vertex-edge graphs and tree diagrams.
    - Can a picture or a vertex-edge graph be drawn with a single line? (degree of vertex)
    - Can you get from any vertex to any other vertex? (connectedness)
3. Use vertex-edge graphs to find solutions to practical problems.
    - Delivery route that stops at specified sites but involves least travel
    - Shortest route from one site on a map to another

Building upon knowledge and skills gained in preceding grades, by the end of Grade 7, students will:

A. Data Analysis

1. Select and use appropriate representations for sets of data, and measures of central tendency (mean, median, and mode).
    - Type of display most appropriate for given data
    - Box-and-whisker plot, upper quartile, lower quartile
    - Scatter plot
    - Calculators and computer used to record and process information
2. Make inferences and formulate and evaluate arguments based on displays and analysis of data.

B. Probability

1. Interpret probabilities as ratios, percents, and decimals.
2. Model situations involving probability with simulations (using spinners, dice, calculators and computers) and theoretical models.
    - Frequency, relative frequency
3. Estimate probabilities and make predictions based on experimental and theoretical probabilities.
4. Play and analyze probability-based games, and discuss the concepts of fairness and expected value.

C. Discrete Mathematics—Systematic Listing and Counting

1. Apply the multiplication principle of counting.
    - Permutations: ordered situations with replacement (e.g., number of possible license plates) vs. ordered situations without replacement (e.g., number of possible slates of 3 class officers from a 23 student class)

2. Explore counting problems involving Venn diagrams with three attributes (e.g., there are 15, 20, and 25 students respectively in the chess club, the debating team, and the engineering society; how many different students belong to the three clubs if there are 6 students in chess and debating, 7 students in chess and engineering, 8 students in debating and engineering, and 2 students in all three?).
3. Apply techniques of systematic listing, counting, and reasoning in a variety of different contexts.

D. Discrete Mathematics—Vertex-Edge Graphs and Algorithms

1. Use vertex-edge graphs to represent and find solutions to practical problems.
    - Finding the shortest network connecting specified sites
    - Finding the shortest route on a map from one site to another
    - Finding the shortest circuit on a map that makes a tour of specified sites

Building upon knowledge and skills gained in preceding grades, by the end of Grade 8, students will:

A. Data Analysis

1. Select and use appropriate representations for sets of data, and measures of central tendency (mean, median, and mode).
    - Type of display most appropriate for given data
    - Box-and-whisker plot, upper quartile, lower quartile
    - Scatter plot
    - Calculators and computer used to record and process information
    - Finding the median and mean (weighted average) using frequency data.
    - Effect of additional data on measures of central tendency
2. Make inferences and formulate and evaluate arguments based on displays and analysis of data.
3. Estimate lines of best fit and use them to interpolate within the range of the data.
4. Use surveys and sampling techniques to generate data and draw conclusions about large groups.

B. Probability

1. Interpret probabilities as ratios, percents, and decimals.
2. Determine probabilities of compound events.
3. Explore the probabilities of conditional events (e.g., if there are seven marbles in a bag, three red and four green, what is the probability that two marbles picked from the bag, without replacement, are both red).
4. Model situations involving probability with simulations (using spinners, dice, calculators and computers) and theoretical models.
    Frequency, relative frequency
5. Estimate probabilities and make predictions based on experimental and theoretical probabilities.
6. Play and analyze probability-based games, and discuss the concepts of fairness and expected value.

C. Discrete Mathematics—Systematic Listing and Counting

1. Apply the multiplication principle of counting.
    - Permutations: ordered situations with replacement (e.g., number of possible license plates) vs. ordered situations without replacement (e.g., number of possible slates of 3 class officers from a 23 student class)
    - Factorial notation
    - Concept of combinations (e.g., number of possible delegations of 3 out of 23 students)
2. Explore counting problems involving Venn diagrams with three attributes (e.g., there are 15, 20, and 25 students respectively in the chess club, the debating team, and the engineering society; how many different students belong to the three clubs if there are 6 students in chess and debating, 7 students in chess and engineering, 8 students in debating and engineering, and 2 students in all three?).
3. Apply techniques of systematic listing, counting, and reasoning in a variety of different contexts.

D. Discrete Mathematics—Vertex-Edge Graphs and Algorithms

1. Use vertex-edge graphs and algorithmic thinking to represent and find solutions to practical problems.
    - Finding the shortest network connecting specified sites
    - Finding a minimal route that includes every street (e.g., for trash pick-up)
    - Finding the shortest route on a map from one site to another
    - Finding the shortest circuit on a map that makes a tour of specified sites
    - Limitations of computers (e.g., the number of routes for a delivery truck visiting *n* sites is *n!*, so finding the shortest circuit by examining all circuits would overwhelm the capacity of any computer, now or in the future, even if n is less than 100)

Building upon knowledge and skills gained in preceding grades, by the end of Grade 12, students will:

A. Data Analysis

1. Use surveys and sampling techniques to generate data and draw conclusions about large groups.
    - Advantages/disadvantages of sample selection methods (e.g., convenience sampling, responses to survey, random sampling)

2. Evaluate the use of data in real-world contexts.
    - Accuracy and reasonableness of conclusions drawn
    - Bias in conclusions drawn (e.g., influence of how data is displayed)
    - Statistical claims based on sampling
3. Design a statistical experiment, conduct the experiment, and interpret and communicate the outcome.
4. Estimate or determine lines of best fit (or curves of best fit if appropriate) with technology, and use them to interpolate within the range of the data.
5. Analyze data using technology, and use statistical terminology to describe conclusions.
    - Measures of dispersion: variance, standard deviation, outliers
    - Correlation coefficient
    - Normal distribution (e.g., approximately 95% of the sample lies between two standard deviations on either side of the mean)

B. Probability

1. Calculate the expected value of a probability-based game, given the probabilities and payoffs of the various outcomes, and determine whether the game is fair.
2. Use concepts and formulas of area to calculate geometric probabilities.
3. Model situations involving probability with simulations (using spinners, dice, calculators and computers) and theoretical models, and solve problems using these models.
4. Determine probabilities in complex situations.
    - Conditional events
    - Complementary events
    - Dependent and independent events
5. Estimate probabilities and make predictions based on experimental and theoretical probabilities.
6. Understand and use the "law of large numbers" (that experimental results tend to approach theoretical probabilities after a large number of trials).

C. Discrete Mathematics—Systematic Listing and Counting

1. Calculate combinations with replacement (e.g., the number of possible ways of tossing a coin 5 times and getting 3 heads) and without replacement (e.g., number of possible delegations of 3 out of 23 students).
2. Apply the multiplication rule of counting in complex situations, recognize the difference between situations with replacement and without replacement, and recognize the difference between ordered and unordered counting situations.
3. Justify solutions to counting problems.
4. Recognize and explain relationships involving combinations and Pascal's Triangle, and apply those methods to situations involving probability.

D. Discrete Mathematics—Vertex-Edge Graphs and Algorithms

1. Use vertex-edge graphs and algorithmic thinking to represent and solve practical problems.
    - Circuits that include every edge in a graph
    - Circuits that include every vertex in a graph
    - Scheduling problems (e.g., when project meetings should be scheduled to avoid conflicts) using graph coloring
    - Applications to science (e.g., who-eats-whom graphs, genetic trees, molecular structures)

2. Explore strategies for making fair decisions.
    - Combining individual preferences into a group decision (e.g., determining winner of an election or selection process)
    - Determining how many Student Council representatives each class (9^{th}, 10^{th}, 11^{th}, and 12^{th} grade) gets when the classes have unequal sizes (apportionment)

STANDARD 4.5 (MATHEMATICAL PROCESSES) ALL STUDENTS WILL USE MATHEMATICAL PROCESSES OF PROBLEM SOLVING, COMMUNICATION, CONNECTIONS, REASONING, REPRESENTATIONS, AND TECHNOLOGY TO SOLVE PROBLEMS AND COMMUNICATE MATHEMATICAL IDEAS.

Descriptive Statement: The mathematical processes described here highlight ways of acquiring and using the content knowledge and skills delineated in the first four mathematics standards.

Problem Solving. Problem posing and problem solving involve examining situations that arise in mathematics and other disciplines and in common experiences, describing these situations mathematically, formulating appropriate mathematical questions, and using a variety of strategies to find solutions. Through problem solving, students experience the power and usefulness of mathematics. Problem solving is interwoven throughout the grades to provide a context for learning and applying mathematical ideas.

Communication. Communication of mathematical ideas involves students' sharing their mathematical understandings in oral and written form with their classmates, teachers, and parents. Such communication helps students clarify and solidify their understanding of mathematics and develop confidence in themselves as mathematics learners. It also enables teachers to better monitor student progress.

Connections. Making connections involves seeing relationships between different topics, and drawing on those relationships in future study. This applies within mathematics, so that students can translate readily between fractions and decimals, or between algebra and geometry; to other content areas, so that students understand how mathematics is used in the sciences, the social sciences, and the arts; and to the everyday world, so that students can connect school mathematics to daily life.

Reasoning. Mathematical reasoning is the critical skill that enables a student to make use of all other mathematical skills. With the development of mathematical reasoning, students recognize that mathematics makes sense and can be understood. They learn how to evaluate situations, select problem-solving strategies, draw logical conclusions, develop and describe solutions, and recognize how those solutions can be applied.

Representations. Representations refers to the use of physical objects, drawings, charts, graphs, and symbols to represent mathematical concepts and problem situations. By using various representations, students will be better able to communicate their thinking and solve problems. Using multiple representations will enrich the problem solver with alternative perspectives on the problem. Historically, people have developed and successfully used manipulatives (concrete representations such as fingers, base ten blocks, geoboards, and algebra tiles) and other representations (such as coordinate systems) to help them understand and develop mathematics.

Technology. Calculators and computers need to be used along with other mathematical tools by students in both instructional and assessment activities. These tools should be used, not to replace mental math and paper-and-pencil computational skills, but to enhance understanding of mathematics and the power to use mathematics. Students should explore both new and familiar concepts with calculators and computers and should also become proficient in using technology as it is used by adults (e.g., for assistance in solving real-world problems).

Strands and Cumulative Progress Indicators

At each grade level, with respect to content appropriate for that grade level, students will:

A. Problem Solving

1. Learn mathematics through problem solving, inquiry, and discovery.
2. Solve problems that arise in mathematics and in other contexts (cf. workplace readiness standard 8.3).
    - Open-ended problems
    - Non-routine problems
    - Problems with multiple solutions
    - Problems that can be solved in several ways
3. Select and apply a variety of appropriate problem-solving strategies (e.g., "try a simpler problem" or "make a diagram") to solve problems.
4. Pose problems of various types and levels of difficulty.
5. Monitor their progress and reflect on the process of their problem solving activity.

B. Communication

1. Use communication to organize and clarify their mathematical thinking.
    - Reading and writing
    - Discussion, listening, and questioning
2. Communicate their mathematical thinking coherently and clearly to peers, teachers, and others, both orally and in writing.
3. Analyze and evaluate the mathematical thinking and strategies of others.
4. Use the language of mathematics to express mathematical ideas precisely.

C. Connections

1. Recognize recurring themes across mathematical domains (e.g., patterns in number, algebra, and geometry).
2. Use connections among mathematical ideas to explain concepts (e.g., two linear equations have a unique solution because the lines they represent intersect at a single point).
3. Recognize that mathematics is used in a variety of contexts outside of mathematics.
4. Apply mathematics in practical situations and in other disciplines.
5. Trace the development of mathematical concepts over time and across cultures (cf. world languages and social studies standards).
6. Understand how mathematical ideas interconnect and build on one another to produce a coherent whole.

D. Reasoning

1. Recognize that mathematical facts, procedures, and claims must be justified.
2. Use reasoning to support their mathematical conclusions and problem solutions.
3. Select and use various types of reasoning and methods of proof.
4. Rely on reasoning, rather than answer keys, teachers, or peers, to check the correctness of their problem solutions.

5. Make and investigate mathematical conjectures.
    - Counterexamples as a means of disproving conjectures
    - Verifying conjectures using informal reasoning or proofs.
6. Evaluate examples of mathematical reasoning and determine whether they are valid.

E. Representations

1. Create and use representations to organize, record, and communicate mathematical ideas.
    - Concrete representations (e.g., base-ten blocks or algebra tiles)
    - Pictorial representations (e.g., diagrams, charts, or tables)
    - Symbolic representations (e.g., a formula)
    - Graphical representations (e.g., a line graph)
2. Select, apply, and translate among mathematical representations to solve problems.
3. Use representations to model and interpret physical, social, and mathematical phenomena.

F. Technology

1. Use technology to gather, analyze, and communicate mathematical information.
2. Use computer spreadsheets, software, and graphing utilities to organize and display quantitative information.
3. Use graphing calculators and computer software to investigate properties of functions and their graphs.
4. Use calculators as problem-solving tools (e.g., to explore patterns, to validate solutions).
5. Use computer software to make and verify conjectures about geometric objects.
6. Use computer-based laboratory technology for mathematical applications in the sciences.