OCEAN COUNTY MATHEMATICS CURRICULUM
SOUTHERN REGIONAL SCHOOL DISTRICT

Content Area: Mathematics

Course Title: Calculus II Advanced Placement BC	**Grade Level: 12**

Unit Plan 1 **Review of Calculus AB topics**	**Pacing Guide** **1 week**
Unit Plan 2 **The Definite Integral**	**Pacing Guide** **2 weeks**
Unit Plan 3 **Differential Equations and Mathematical Modeling**	**Pacing Guide** **5-6 weeks**
Unit Plan 4 **Applications of Definite Integrals**	**Pacing Guide** **3 weeks**
Unit Plan 5 **Essentials of Limits**	**Pacing Guide** **1 week**
Unit Plan 6 **L'Hopital's Rule**	**Pacing Guide** **2 week**
Unit Plan 7 **The Derivative of a Function**	**Pacing Guide** **2 weeks**

Unit Plan	Pacing Guide
Unit Plan 8 **Infinite Series**	**Pacing Guide** **6 weeks**
Unit Plan 9 **Applications of the Derivative**	**Pacing Guide** **2 weeks**
Unit Plan 10 **Parametric, Vector and Polar Functions**	**Pacing Guide** **5 weeks**
Unit Plan 11 **Preparation for AP Test**	**Pacing Guide** **3 weeks**

Date Created:	**March 2012**
Board Approved on:	**March 14, 2012**

OCEAN COUNTY MATHEMATICS CURRICULUM
SOUTHERN REGIONAL SCHOOL DISTRICT
Unit Overview

Content Area: Mathematics

Unit Title: Review of Calculus AB topics

Target Course/Grade Level: Calculus II Advanced Placement BC / 12

Unit Summary
Students will develop an understanding for the five main topics to be included in the Calculus II AP course. Students will not develop proficiency in any of the topics. It is simply a quick five-day walk through the main topics of the course.

Primary interdisciplinary connections:
Infused within the unit are connection to the 2009 NJCCCS for Mathematics, Language Arts Literacy and Technology.

21st century themes:
The unit will integrate the 21st Century Life and Career stand 9.1 strands A-D. These strands include: Critical thinking and problem solving, creativity and innovation, collaboration, teamwork and leadership, and cross cultural understanding and interpersonal communication.

Technology connections:
For further clarification refer to NJ Class Standard Introductions at http://www.state.nj.us/education/cccs/

Learning Targets

Content Standards
Interpreting Functions F-IF
Understand the concept of a function and use function notation

1. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.
2. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
3. Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. For *example, the Fibonacci sequence is defined recursively by* $f(0) = f(1) = 1$, $f(n+1) = f(n) + f(n-1)$ *for* $n \geq 1$.

Interpret functions that arise in applications in terms of the context

4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*
5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*★
6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

Analyze functions using different representations

7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
   a. Graph linear and quadratic functions and show intercepts, maxima, and minima.
   b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.

c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
d. (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.
e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
   a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.
   b. Use the properties of exponents to interpret expressions for exponential functions. *For example, identify percent rate of change in functions such as y = (1.02)t, y = (0.97)t, y = (1.01)12t, y = (1.2)t/10, and classify them as representing exponential growth or decay.*
9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*

Building Functions F-BF

Build a function that models a relationship between two quantities

1. Write a function that describes a relationship between two quantities.
   a. Determine an explicit expression, a recursive process, or steps for calculation from a context.
   b. Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.*
   c. (+) Compose functions. *For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.*
2. Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.

Build new functions from existing functions

3. Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.*
4. Find inverse functions.
   a. Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. *For example, f(x) =2 x3 or f(x) = (x+1)/(x–1) for x ≠ 1.*
   b. (+) Verify by composition that one function is the inverse of another.
   c. (+) Read values of an inverse function from a graph or a table, given that the function has an inverse.
   d. (+) Produce an invertible function from a non-invertible function by restricting the domain.
5. (+) Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.

Linear, Quadratic, and Exponential Models★F -LE

Construct and compare linear, quadratic, and exponential models and solve problems

1. Distinguish between situations that can be modeled with linear functions and with exponential functions.
   a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.
   b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
   c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.
2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
3. Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.

4. For exponential models, express as a logarithm the solution to *ab*ct = *d* where *a*, *c*, and *d* are numbers and the base *b* is 2, 10, or *e*; evaluate the logarithm using technology.

Interpret expressions for functions in terms of the situation they model

5. Interpret the parameters in a linear or exponential function in terms of a context.

Trigonometric Functions F-TF

Extend the domain of trigonometric functions using the unit circle

1. Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
2. Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.
3. (+) Use special triangles to determine geometrically the values of sine, cosine, tangent for □/3, □/4 and □/6, and use the unit circle to express the values of sine, cosine, and tangent for □–*x*, □+*x*, and 2□–*x* in terms of their values for *x*, where *x* is any real number.
4. (+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.

Model periodic phenomena with trigonometric functions

5. Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.
6. (+) Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.
7. (+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.

Prove and apply trigonometric identities

8. Prove the Pythagorean identity sin2(θ) + cos2(θ) = 1 and use it to find sin(θ), cos(θ), or tan(θ) given sin(θ), cos(θ), or tan(θ) and the quadrant of the angle.
9. (+) Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.

Unit Essential Questions

- What is Instantaneous Rate of Change?
- How can an equation, a graph or a table determine Rate of Change?
- What is an Integral of a Function?
- How can definite integrals be approximated using Trapezoids?
- What is a limit?

Unit Enduring Understandings

Students will understand that…

- Students will develop an understanding for the five main topics to be included in the Calculus I AP course. Students will not develop proficiency in any of the topics. It is simply a quick five-day walk through the main topics of the course.

Unit Objectives

Students will know…

- The purpose of this unit to give the students an overview of what calculus is all about. Many students have not seen the big picture prior to studying a class. This walk through will give the students an exposure to what lies ahead in their study of Calculus.

Unit Objectives

Students will be able to…

- Have an overview of what calculus is all about. Many students have not seen the big picture prior to studying a class. This walk through will give the students an exposure to what lies ahead in their study of Calculus.

OCEAN COUNTY MATHEMATICS CURRICULUM
SOUTHERN REGIONAL SCHOOL DISTRICT
Evidence of Learning

Formative Assessments
For additional ideas please refer to NJ State DOE classroom application documents:

- Observation
- Homework
- Class participation

Summative Assessments
For additional ideas please refer to NJ State Academic website
http://www.state.nj.us/education/cccs/

- Chapter/Unit Test
- Quizzes
- Presentations
- Unit Projects
- Quarterlies and Final Exams

Modifications (ELLs, Special Education, Gifted and Talented)

- Teacher tutoring
- Peer tutoring
- Cooperative learning groups
- Modified assignments
- Differentiated instruction
- Native language texts and native language to English dictionary
- Follow all IEP modifications/504 plan

Curriculum development Resources/Instructional Materials/Equipment Needed Teacher Resources:

- Textbook: Calculus Graphical, Numerical, Algebraic

Teacher Notes:

OCEAN COUNTY MATHEMATICS CURRICULUM
SOUTHERN REGIONAL SCHOOL DISTRICT
Unit Overview

Content Area: Mathematics

Unit Title: The Definite Integral

Target Course/Grade Level: Calculus II Advanced Placement BC / 12

Unit Summary
Definite integrals can be used to find many real quantities. This unit will prepare students to solving those problems.

Primary interdisciplinary connections:
Infused within the unit are connection to the 2009 NJCCCS for Mathematics, Language Arts Literacy and Technology.

21st century themes:
The unit will integrate the 21st Century Life and Career stand 9.1 strands A-D. These strands include: Critical thinking and problem solving, creativity and innovation, collaboration, teamwork and leadership, and cross cultural understanding and interpersonal communication.

Technology connections:
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Learning Targets

Content Standards
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5. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
6. Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. For *example, the Fibonacci sequence is defined recursively by f(0) = f(1) = 1, f(n+1) = f(n) +f(n-1) for n ≥□1.*

Interpret functions that arise in applications in terms of the context
4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*
5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*★
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9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*

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Trigonometric Functions F-TF

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3. (+) Use special triangles to determine geometrically the values of sine, cosine, tangent for π/3, π/4 and π/6, and use the unit circle to express the values of sine, cosine, and tangent for π–*x*, π+*x*, and 2π–*x* in terms of their values for *x*, where *x* is any real number.
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9. (+) Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.

Unit Essential Questions	**Unit Enduring Understandings** *Students will understand that…*
• What is an antiderivative and what is its relationship to a derivative? • How can definite integrals be calculated and approximated? • What are Riemann Sums and how can they be used to find a definite integral?	• The inter-relationship between area and integrals. • To apply these ideas to find calculated and approximate values of an integral.
Unit Objectives *Students will know…* • Estimating Finite Sums. • Definite Integrals. • Fundamental Theorem of Calculus. • Trapezoidal Rule.	**Unit Objectives** *Students will be able to…* • Find antiderivatives and definite integrals. • Use the Fundamental Theorem of Calculus. • To approximate definite integrals.

OCEAN COUNTY MATHEMATICS CURRICULUM
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Evidence of Learning

Formative Assessments

For additional ideas please refer to NJ State DOE classroom application documents:

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- Homework
- Class participation

Summative Assessments

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- Chapter/Unit Test
- Quizzes
- Presentations
- Unit Projects
- Quarterlies and Final Exams

Modifications (ELLs, Special Education, Gifted and Talented)

- Teacher tutoring
- Peer tutoring
- Cooperative learning groups
- Modified assignments
- Differentiated instruction
- Native language texts and native language to English dictionary
- Follow all IEP modifications/504 plan

Curriculum development Resources/Instructional Materials/Equipment Needed Teacher Resources:

- Textbook: Calculus Graphical, Numerical, Algebraic

Teacher Notes:

OCEAN COUNTY MATHEMATICS CURRICULUM
SOUTHERN REGIONAL SCHOOL DISTRICT
Unit Overview

Content Area: Mathematics

Unit Title: Differential Equations and Mathematical Modeling

Target Course/Grade Level: Calculus II Advanced Placement BC / 12

Unit Summary
The techniques taught in this unit help the students be able to use differential equations to predict future position given a current position and velocity of a particle by looking at analytical, graphical, and numerical methods that lead to these solutions.

Primary interdisciplinary connections:
Infused within the unit are connection to the 2009 NJCCCS for Mathematics, Language Arts Literacy and Technology.

21st century themes:
The unit will integrate the 21st Century Life and Career stand 9.1 strands A-D. These strands include: Critical thinking and problem solving, creativity and innovation, collaboration, teamwork and leadership, and cross cultural understanding and interpersonal communication.

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Content Standards
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8. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
9. Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. For *example, the Fibonacci sequence is defined recursively by f(0) = f(1) = 1, f(n+1) = f(n) +f(n-1) for n ≥□1.*

Interpret functions that arise in applications in terms of the context

4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*
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increasing linearly, quadratically, or (more generally) as a polynomial function.

4. For exponential models, express as a logarithm the solution to *ab*ct = *d* where *a*, *c*, and *d* are numbers and the base *b* is 2, 10, or *e*; evaluate the logarithm using technology.

Interpret expressions for functions in terms of the situation they model

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Trigonometric Functions F-TF

Extend the domain of trigonometric functions using the unit circle

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Prove and apply trigonometric identities

8. Prove the Pythagorean identity sin2(θ) + cos2(θ) = 1 and use it to find sin(θ), cos(θ), or tan(θ) given sin(θ), cos(θ), or tan(θ) and the quadrant of the angle.
9. (+) Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.

Unit Essential Questions

- How can differential equations be used to predict future position given a current position and velocity of a particle.
- What are some of the analytical, graphical, and numerical methods that lead to these solutions?

Unit Enduring Understandings

Students will understand that…

- Differential equations can be used to predict future positions given a current position and velocity of an object.
- There are a variety of ways to integrate a function, and will know the appropriate use each method.

Unit Objectives

Students will know…

- Solving initial value problems.
- Antiderivatives and Indefinite Integrals.
- Properties of Indefinite Integrals.
- Power Rule for Integral Form.
- Trigonometric Integrands.
- Substitution in Indefinite Integrals.
- Substitution in Definite Integrals.
- Separable Differential Equations.
- Product Rule in Integral Form.
- Repeated Use of Integration by Parts.
- Tabular Method for Integration by parts.
- Law of Exponential Change.
- Continuous compounded interest.

Unit Objectives

Students will be able to…

- Construct antiderivatives using the Fundamental Theorem of Calculus.
- Find antiderivatives of polynomials, exponential, and selected trigonometric functions of kx.
- Solve initial value problems.
- Compute indefinite and definite integrals by substitution.
- Solve differential equations by separating the variables.
- Use integration by parts to evaluate indefinite and definite integrals
- Use the tabular method for integration by parts.
- Solve problems involving exponential growth and decay.
- Solve problems involving exponential and logistic population growth.

• Radioactivity. • Newton's Law of Cooling. • Resistance Proportional to Velocity. • Exponential Model. • Logistic Growth Model. • Logistic Regression. • Euler's Method. • Numerical Solutions. • Graphical Solutions. • Improved Euler's Method	• Use Euler's Method and the improved Euler's Method to find approximate solutions to initial value problems.

OCEAN COUNTY MATHEMATICS CURRICULUM
SOUTHERN REGIONAL SCHOOL DISTRICT
Evidence of Learning

Formative Assessments
For additional ideas please refer to NJ State DOE classroom application documents:

- Observation
- Homework
- Class participation

Summative Assessments
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http://www.state.nj.us/education/cccs/

- Chapter/Unit Test
- Quizzes
- Presentations
- Unit Projects
- Quarterlies and Final Exams

Modifications (ELLs, Special Education, Gifted and Talented)

- Teacher tutoring
- Peer tutoring
- Cooperative learning groups
- Modified assignments
- Differentiated instruction
- Native language texts and native language to English dictionary
- Follow all IEP modifications/504 plan

Curriculum development Resources/Instructional Materials/Equipment Needed Teacher Resources:

- Textbook: Calculus Graphical, Numerical, Algebraic

Teacher Notes:

OCEAN COUNTY MATHEMATICS CURRICULUM
SOUTHERN REGIONAL SCHOOL DISTRICT
Unit Overview

Content Area: Mathematics

Unit Title: Applications of Definite Integrals

Target Course/Grade Level: Calculus II Advanced Placement BC / 12

Unit Summary
This unit provides students with techniques to deal with the area under a function

Primary interdisciplinary connections:
Infused within the unit are connection to the 2009 NJCCCS for Mathematics, Language Arts Literacy and Technology.

21st century themes:
The unit will integrate the 21st Century Life and Career stand 9.1 strands A-D. These strands include: Critical thinking and problem solving, creativity and innovation, collaboration, teamwork and leadership, and cross cultural understanding and interpersonal communication.

Technology connections:
For further clarification refer to NJ Class Standard Introductions at http://www.state.nj.us/education/cccs/

Learning Targets

Content Standards
Interpreting Functions F-IF
Understand the concept of a function and use function notation

10. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If *f* is a function and *x* is an element of its domain, then *f*(*x*) denotes the output of *f* corresponding to the input *x*. The graph of *f* is the graph of the equation *y* = *f*(*x*).
11. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
12. Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. For *example, the Fibonacci sequence is defined recursively by f(0) = f(1) = 1, f(n+1) = f(n) +f(n-1) for n ≥ 1.*

Interpret functions that arise in applications in terms of the context

4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*
5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*★
6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

Analyze functions using different representations

7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
   p. Graph linear and quadratic functions and show intercepts, maxima, and minima.
   q. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.

r. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
s. (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.
t. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
   g. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.
   h. Use the properties of exponents to interpret expressions for exponential functions. *For example, identify percent rate of change in functions such as y = (1.02)t, y = (0.97)t, y = (1.01)12t, y = (1.2)t/10, and classify them as representing exponential growth or decay.*
9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*

Building Functions F-BF

Build a function that models a relationship between two quantities

1. Write a function that describes a relationship between two quantities.
   j. Determine an explicit expression, a recursive process, or steps for calculation from a context.
   k. Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.*
   l. (+) Compose functions. *For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.*
2. Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.

Build new functions from existing functions

3. Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.*
4. Find inverse functions.
   m. Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. *For example, f(x) =2 x3 or f(x) = (x+1)/(x–1) for x ≠ 1.*
   n. (+) Verify by composition that one function is the inverse of another.
   o. (+) Read values of an inverse function from a graph or a table, given that the function has an inverse.
   p. (+) Produce an invertible function from a non-invertible function by restricting the domain.
5. (+) Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.

Linear, Quadratic, and Exponential Models★F -LE

Construct and compare linear, quadratic, and exponential models and solve problems

1. Distinguish between situations that can be modeled with linear functions and with exponential functions.
   j. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.
   k. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
   l. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.
2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
3. Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.

4. For exponential models, express as a logarithm the solution to *ab*ct = *d* where *a*, *c*, and *d* are numbers and the base *b* is 2, 10, or *e*; evaluate the logarithm using technology.

Interpret expressions for functions in terms of the situation they model

5. Interpret the parameters in a linear or exponential function in terms of a context.

Trigonometric Functions F-TF

Extend the domain of trigonometric functions using the unit circle

1. Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
2. Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.
3. (+) Use special triangles to determine geometrically the values of sine, cosine, tangent for □/3, □/4 and □/6, and use the unit circle to express the values of sine, cosine, and tangent for □–*x*, □+*x*, and 2□–*x* in terms of their values for *x*, where *x* is any real number.
4. (+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.

Model periodic phenomena with trigonometric functions

5. Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.
6. (+) Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.
7. (+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.

Prove and apply trigonometric identities

8. Prove the Pythagorean identity sin2(θ) + cos2(θ) = 1 and use it to find sin(θ), cos(θ), or tan(θ) given sin(θ), cos(θ), or tan(θ) and the quadrant of the angle.
9. (+) Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.

Unit Essential Questions

- How is it possible to find exact answers to real problems involving the concepts of area and volume?
- When exact answers are not possible is there a technique that can be used to approximate these areas and volumes?

Unit Enduring Understandings

Students will understand that…

- Integrals can be used to solve problems associated with the concepts of area and volume.

Unit Objectives

Students will know…

- Integral as net change.
- Areas in the plane.
- Volumes.
- Lengths of curves.
- Applications from Science and Statistics.

Unit Objectives

Students will be able to…

- Approximate the area under the graph of a nonnegative continuous function using rectangle approximations methods.
- Interpret the area under a graph as a net accumulations of a rate of change.
- Express the area under a graph as a definite integral and as a limit of Riemann Sums.
- Compute the area under a curve using numerical integration procedures.
- Apply the rules of definite integrals and find the average value of a function over a closed interval.
- Apply the Fundamental Theorem of Calculus.
- Understand the relationship between the derivative and the definite integral as expressed in both parts of the Fundamental Theorem of Calculus.
- Approximate the definite integral by using the Trapezoidal Rule and by using Simpson’s Rule, and estimate the error in using the Trapezoidal and Simpson’s Rule.

OCEAN COUNTY MATHEMATICS CURRICULUM
SOUTHERN REGIONAL SCHOOL DISTRICT
Evidence of Learning

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- Teacher tutoring
- Peer tutoring
- Cooperative learning groups
- Modified assignments
- Differentiated instruction
- Native language texts and native language to English dictionary
- Follow all IEP modifications/504 plan

Curriculum development Resources/Instructional Materials/Equipment Needed Teacher Resources:

- Textbook: Calculus Graphical, Numerical, Algebraic

Teacher Notes:

OCEAN COUNTY MATHEMATICS CURRICULUM
SOUTHERN REGIONAL SCHOOL DISTRICT
Unit Overview

Content Area: Mathematics

Unit Title: Essentials of Limits

Target Course/Grade Level: Calculus II Advanced Placement BC / 12

Unit Summary

To develop an understanding for the existence of a limit and how limits are found and proven. This is a foundation for the rest of the calculus course.

Primary interdisciplinary connections:

Infused within the unit are connection to the 2009 NJCCCS for Mathematics, Language Arts Literacy and Technology.

21st century themes:

The unit will integrate the 21st Century Life and Career stand 9.1 strands A-D. These strands include: Critical thinking and problem solving, creativity and innovation, collaboration, teamwork and leadership, and cross cultural understanding and interpersonal communication.

Technology connections:

For further clarification refer to NJ Class Standard Introductions at http://www.state.nj.us/education/cccs/

Learning Targets

Content Standards

Interpreting Functions F-IF

Understand the concept of a function and use function notation

13. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If *f* is a function and *x* is an element of its domain, then *f*(*x*) denotes the output of *f* corresponding to the input *x*. The graph of *f* is the graph of the equation *y* = *f*(*x*).
14. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
15. Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. For *example, the Fibonacci sequence is defined recursively by f(0) = f(1) = 1, f(n+1) = f(n) +f(n-1) for n ≥ 1.*

Interpret functions that arise in applications in terms of the context

4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*
5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*★
6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

Analyze functions using different representations

7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
   u. Graph linear and quadratic functions and show intercepts, maxima, and minima.
   v. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.

    w. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
    x. (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.
    y. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
    i. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.
    j. Use the properties of exponents to interpret expressions for exponential functions. *For example, identify percent rate of change in functions such as $y = (1.02)t$, $y = (0.97)t$, $y = (1.01)12t$, $y = (1.2)t/10$, and classify them as representing exponential growth or decay.*
9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*

Building Functions F-BF

Build a function that models a relationship between two quantities

1. Write a function that describes a relationship between two quantities.
    m. Determine an explicit expression, a recursive process, or steps for calculation from a context.
    n. Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.*
    o. (+) Compose functions. *For example, if $T(y)$ is the temperature in the atmosphere as a function of height, and $h(t)$ is the height of a weather balloon as a function of time, then $T(h(t))$ is the temperature at the location of the weather balloon as a function of time.*
2. Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.

Build new functions from existing functions

3. Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.*
4. Find inverse functions.
    q. Solve an equation of the form $f(x) = c$ for a simple function f that has an inverse and write an expression for the inverse. *For example, $f(x) =2\ x3$ or $f(x) = (x+1)/(x–1)$ for $x \neq 1$.*
    r. (+) Verify by composition that one function is the inverse of another.
    s. (+) Read values of an inverse function from a graph or a table, given that the function has an inverse.
    t. (+) Produce an invertible function from a non-invertible function by restricting the domain.
5. (+) Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.

Linear, Quadratic, and Exponential Models★F -LE

Construct and compare linear, quadratic, and exponential models and solve problems

1. Distinguish between situations that can be modeled with linear functions and with exponential functions.
    m. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.
    n. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
    o. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.
2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
3. Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.

4. For exponential models, express as a logarithm the solution to *ab*ct = *d* where *a*, *c*, and *d* are numbers and the base *b* is 2, 10, or *e*; evaluate the logarithm using technology.

Interpret expressions for functions in terms of the situation they model

5. Interpret the parameters in a linear or exponential function in terms of a context.

Trigonometric Functions F-TF

Extend the domain of trigonometric functions using the unit circle

1. Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
2. Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.
3. (+) Use special triangles to determine geometrically the values of sine, cosine, tangent for □/3, □/4 and □/6, and use the unit circle to express the values of sine, cosine, and tangent for □–*x*, □+*x*, and 2□–*x* in terms of their values for *x*, where *x* is any real number.
4. (+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.

Model periodic phenomena with trigonometric functions

5. Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.
6. (+) Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.
7. (+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.

Prove and apply trigonometric identities

8. Prove the Pythagorean identity sin2(θ) + cos2(θ) = 1 and use it to find sin(θ), cos(θ), or tan(θ) given sin(θ), cos(θ), or tan(θ) and the quadrant of the angle.
9. (+) Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.

Unit Essential Questions

- What is the definition of a limit?
- What are the essential techniques used to find limits?
- How can limits be proven using the ε and δ?
- How does continuity depend on limits?

Unit Enduring Understandings

Students will understand that…

- A limit, technics to finding a limit and how it pertains to continuity.

Unit Objectives

Students will know…

- Definition of limit.
- Rules for finding limits and δ proofs for limits.
- Continuity of a function.

Unit Objectives

Students will be able to…

- Define a limit using the ε and δ definition.
- Find limits using the properties.
- Prove a limit using the ε and δ definition.
- Discuss the continuity of a function.

OCEAN COUNTY MATHEMATICS CURRICULUM
SOUTHERN REGIONAL SCHOOL DISTRICT
Evidence of Learning

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- Teacher tutoring
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Curriculum development Resources/Instructional Materials/Equipment Needed Teacher Resources:

- Textbook: Calculus Graphical, Numerical, Algebraic

Teacher Notes:

OCEAN COUNTY MATHEMATICS CURRICULUM
SOUTHERN REGIONAL SCHOOL DISTRICT
Unit Overview

Content Area: Mathematics

Unit Title: L'Hopital's Rule

Target Course/Grade Level: Calculus II Advanced Placement BC / 12

Unit Summary

L'Hopital's Rule will be used to compare the rates at which functions grow as $|x|$ grows very large - this has not been possible up to this point in the course.
Being able to evaluate improper integrals will fill in the missing techniques for dealing with discontinuous functions and find area and volume with these functions
Partial Fractions will give us the tools for evaluating many rational integrands.

Primary interdisciplinary connections:
Infused within the unit are connection to the 2009 NJCCCS for Mathematics, Language Arts Literacy and Technology.

21st century themes:
The unit will integrate the 21st Century Life and Career stand 9.1 strands A-D. These strands include: Critical thinking and problem solving, creativity and innovation, collaboration, teamwork and leadership, and cross cultural understanding and interpersonal communication.

Technology connections:
For further clarification refer to NJ Class Standard Introductions at http://www.state.nj.us/education/cccs/

Learning Targets

Content Standards
Interpreting Functions F-IF
Understand the concept of a function and use function notation

16. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If *f* is a function and *x* is an element of its domain, then *f*(*x*) denotes the output of *f* corresponding to the input *x*. The graph of *f* is the graph of the equation $y = f(x)$.
17. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
18. Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. For *example, the Fibonacci sequence is defined recursively by* $f(0) = f(1) = 1$, $f(n+1) = f(n) + f(n-1)$ *for* $n \geq 1$.

Interpret functions that arise in applications in terms of the context

4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*
5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*★
6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

Analyze functions using different representations

7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases

and using technology for more complicated cases.

- z. Graph linear and quadratic functions and show intercepts, maxima, and minima.
- aa. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
- bb. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
- cc. (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.
- dd. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
   - k. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.
   - *l.* Use the properties of exponents to interpret expressions for exponential functions. *For example, identify percent rate of change in functions such as y = (1.02)t, y = (0.97)t, y = (1.01)12t, y = (1.2)t/10, and classify them as representing exponential growth or decay.*
9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*

Building Functions F-BF

Build a function that models a relationship between two quantities

1. Write a function that describes a relationship between two quantities.
   - p. Determine an explicit expression, a recursive process, or steps for calculation from a context.
   - *q.* Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.*
   - *r.* (+) Compose functions. *For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.*
2. Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.

Build new functions from existing functions

3. Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.*
4. Find inverse functions.
   - *u.* Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. *For example, f(x) =2 x3 or f(x) = (x+1)/(x–1) for x ≠ □1.*
   - v. (+) Verify by composition that one function is the inverse of another.
   - w. (+) Read values of an inverse function from a graph or a table, given that the function has an inverse.
   - x. (+) Produce an invertible function from a non-invertible function by restricting the domain.
5. (+) Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.

Linear, Quadratic, and Exponential Models★F -LE

Construct and compare linear, quadratic, and exponential models and solve problems

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   - p. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.
   - q. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
   - r. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
3. Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.
4. For exponential models, express as a logarithm the solution to *ab*ct = *d* where *a*, *c*, and *d* are numbers and the base *b* is 2, 10, or *e*; evaluate the logarithm using technology.

Interpret expressions for functions in terms of the situation they model

5. Interpret the parameters in a linear or exponential function in terms of a context.

Trigonometric Functions F-TF

Extend the domain of trigonometric functions using the unit circle

1. Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
2. Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.
3. (+) Use special triangles to determine geometrically the values of sine, cosine, tangent for □/3, □/4 and □/6, and use the unit circle to express the values of sine, cosine, and tangent for □–*x*, □+*x*, and 2□–*x* in terms of their values for *x*, where *x* is any real number.
4. (+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.

Model periodic phenomena with trigonometric functions

5. Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.
6. (+) Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.
7. (+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.

Prove and apply trigonometric identities

8. Prove the Pythagorean identity sin2(θ) + cos2(θ) = 1 and use it to find sin(θ), cos(θ), or tan(θ) given sin(θ), cos(θ), or tan(θ) and the quadrant of the angle.
9. (+) Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.

Unit Essential Questions

- How it is possible to determine limits analytically using concepts of derivatives?
- How can improper integrals (functions that approach infinity as x approaches a particular value) be calculated?
- How can integrals involving many rational functions be re-written so they can be evaluated?

Unit Enduring Understandings

Students will understand that…

- Limits can be used to evaluate improper integrals.
- Partial fractions, integral tables and trig substitution are all methods for evaluating integrals.

Unit Objectives

Students will know…

- L'Hopital's Rule with indeterminate forms 0/0, ∞/∞, $\infty \cdot 0$, $\infty \cdot -\infty$, 1^∞, 0^0, and ∞^0
- Comparing Rates of Growth.
- Order and Oh-Notation.
- Sequential versus Binary Search.
- Infinite limits of integration.
- Partial Fractions and Integral Tables.
- Integrands with Infinite Discontinuities.
- Tests for Convergence and

Unit Objectives

Students will be able to…

- Find limits of indeterminate forms using L'Hopital's Rule.
- Use little-oh and big-oh notation in determining, investigating, and comparing rates of growth of a function.
- Use limits to evaluate improper integrals.
- Use the direct comparison test and the limit comparison test to determine the convergence or divergence of improper integrals.
- Evaluate integrals using partial fractions, integral tables, and trigonometric substitutions.

Divergence. • Partial Fractions. • General Description for the Method of integrating with Partial Fractions. • Integral Tables. • Trigonometric Substitutions.	

OCEAN COUNTY MATHEMATICS CURRICULUM
SOUTHERN REGIONAL SCHOOL DISTRICT
Evidence of Learning

Formative Assessments
For additional ideas please refer to NJ State DOE classroom application documents:

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- Peer tutoring
- Cooperative learning groups
- Modified assignments
- Differentiated instruction
- Native language texts and native language to English dictionary
- Follow all IEP modifications/504 plan

Curriculum development Resources/Instructional Materials/Equipment Needed Teacher Resources:

- Textbook: Calculus Graphical, Numerical, Algebraic

Teacher Notes:

OCEAN COUNTY MATHEMATICS CURRICULUM
SOUTHERN REGIONAL SCHOOL DISTRICT
Unit Overview

Content Area: Mathematics

Unit Title: The Derivative of a Function

Target Course/Grade Level: Calculus II Advanced Placement BC / 12

Unit Summary
To be sure the students have a solid foundation of a derivative graphically, numerically, and analytically.

Primary interdisciplinary connections:
Infused within the unit are connection to the 2009 NJCCCS for Mathematics, Language Arts Literacy and Technology.

21st century themes:
The unit will integrate the 21st Century Life and Career stand 9.1 strands A-D. These strands include: Critical thinking and problem solving, creativity and innovation, collaboration, teamwork and leadership, and cross cultural understanding and interpersonal communication.

Technology connections:
For further clarification refer to NJ Class Standard Introductions at http://www.state.nj.us/education/cccs/

Learning Targets

Content Standards
Interpreting Functions F-IF
Understand the concept of a function and use function notation
19. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.
20. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
21. Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. For *example, the Fibonacci sequence is defined recursively by* $f(0) = f(1) = 1$, $f(n+1) = f(n) + f(n-1)$ *for* $n \geq$ □ *1.*

Interpret functions that arise in applications in terms of the context
4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*
5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*★
6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

Analyze functions using different representations
7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
   - ee. Graph linear and quadratic functions and show intercepts, maxima, and minima.
   - ff. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
   - gg. Graph polynomial functions, identifying zeros when suitable factorizations are available, and

showing end behavior.

hh. (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.

ii. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

    m. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.

    n. Use the properties of exponents to interpret expressions for exponential functions. *For example, identify percent rate of change in functions such as y = (1.02)t, y = (0.97)t, y = (1.01)12t, y = (1.2)t/10, and classify them as representing exponential growth or decay.*

9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*

Building Functions F-BF

Build a function that models a relationship between two quantities

1. Write a function that describes a relationship between two quantities.

    s. Determine an explicit expression, a recursive process, or steps for calculation from a context.

    t. Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.*

    u. (+) Compose functions. *For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.*

2. Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.

Build new functions from existing functions

3. Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.*

4. Find inverse functions.

    y. Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. *For example, f(x) =2 x3 or f(x) = (x+1)/(x–1) for x ≠□1.*

    z. (+) Verify by composition that one function is the inverse of another.

    aa. (+) Read values of an inverse function from a graph or a table, given that the function has an inverse.

    bb. (+) Produce an invertible function from a non-invertible function by restricting the domain.

5. (+) Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.

Linear, Quadratic, and Exponential Models★F -LE

Construct and compare linear, quadratic, and exponential models and solve problems

1. Distinguish between situations that can be modeled with linear functions and with exponential functions.

    s. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.

    t. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.

    u. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

3. Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.

4. For exponential models, express as a logarithm the solution to *ab*ct = *d* where *a*, *c*, and *d* are numbers

and the base *b* is 2, 10, or *e*; evaluate the logarithm using technology.

Interpret expressions for functions in terms of the situation they model

5. Interpret the parameters in a linear or exponential function in terms of a context.

Trigonometric Functions F-TF

Extend the domain of trigonometric functions using the unit circle

1. Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
2. Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.
3. (+) Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and use the unit circle to express the values of sine, cosine, and tangent for $\pi - x$, $\pi + x$, and $2\pi - x$ in terms of their values for x, where x is any real number.
4. (+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.

Model periodic phenomena with trigonometric functions

5. Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.
6. (+) Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.
7. (+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.

Prove and apply trigonometric identities

8. Prove the Pythagorean identity sin2(θ) + cos2(θ) = 1 and use it to find sin(θ), cos(θ), or tan(θ) given sin(θ), cos(θ), or tan(θ) and the quadrant of the angle.
9. (+) Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.

Unit Essential Questions

- How is a derivative defined?
- What is the meaning of a derivative numerically and graphically?
- What are the rules for derivatives?

Unit Enduring Understandings

Students will understand that…

- The rules of derivatives as well as the meaning of a derivative numerically and graphically.

Unit Objectives

Students will know…

- Definition of a derivative.
- Rules for differentiation
- Chain Rule.
- Implicit Differentiation.
- Derivatives of Inverse. Trigonometric Functions.
- Derivatives of Exponential and Logarithmic Functions.

Unit Objectives

Students will be able to…

- To demonstrate a thorough understanding of the definition of a derivative and the rules for working with derivatives.
- To demonstrate a thorough understanding of applying derivatives to inverse trigonometric functions, exponential functions, and logarithmic functions.

OCEAN COUNTY MATHEMATICS CURRICULUM
SOUTHERN REGIONAL SCHOOL DISTRICT
Evidence of Learning

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- Differentiated instruction
- Native language texts and native language to English dictionary
- Follow all IEP modifications/504 plan

Curriculum development Resources/Instructional Materials/Equipment Needed Teacher Resources:

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Teacher Notes:

OCEAN COUNTY MATHEMATICS CURRICULUM
SOUTHERN REGIONAL SCHOOL DISTRICT
Unit Overview

Content Area: Mathematics

Unit Title: Infinite Series

Target Course/Grade Level: Calculus II Advanced Placement BC / 12

Unit Summary
Many functions can be approximated with Taylor or Maclaurin Series. By approximating the functions with these series it is possible to evaluate their derivative and/or integral to study the functions behavior.

Primary interdisciplinary connections:
Infused within the unit are connection to the 2009 NJCCCS for Mathematics, Language Arts Literacy and Technology.

21st century themes:
The unit will integrate the 21st Century Life and Career stand 9.1 strands A-D. These strands include: Critical thinking and problem solving, creativity and innovation, collaboration, teamwork and leadership, and cross cultural understanding and interpersonal communication.

Technology connections:
For further clarification refer to NJ Class Standard Introductions at http://www.state.nj.us/education/cccs/

Learning Targets

Content Standards
Interpreting Functions F-IF
Understand the concept of a function and use function notation

22. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If *f* is a function and *x* is an element of its domain, then *f*(*x*) denotes the output of *f* corresponding to the input *x*. The graph of *f* is the graph of the equation *y* = *f*(*x*).
23. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
24. Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. For *example, the Fibonacci sequence is defined recursively by f(0) = f(1) = 1, f(n+1) = f(n) +f(n-1) for n ≥ 1.*

Interpret functions that arise in applications in terms of the context

4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*
5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*★
6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

Analyze functions using different representations

7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
   - jj. Graph linear and quadratic functions and show intercepts, maxima, and minima.
   - kk. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.

ll. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.

mm. (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.

nn. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
   o. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.
   p. Use the properties of exponents to interpret expressions for exponential functions. *For example, identify percent rate of change in functions such as y = (1.02)t, y = (0.97)t, y = (1.01)12t, y = (1.2)t/10, and classify them as representing exponential growth or decay.*
9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*

Building Functions F-BF

Build a function that models a relationship between two quantities

1. Write a function that describes a relationship between two quantities.
   v. Determine an explicit expression, a recursive process, or steps for calculation from a context.
   w. Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.*
   x. (+) Compose functions. *For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.*
2. Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.

Build new functions from existing functions

3. Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.*
4. Find inverse functions.
   cc. Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. *For example, f(x) =2 x3 or f(x) = (x+1)/(x–1) for x ≠ 1.*
   dd. (+) Verify by composition that one function is the inverse of another.
   ee. (+) Read values of an inverse function from a graph or a table, given that the function has an inverse.
   ff. (+) Produce an invertible function from a non-invertible function by restricting the domain.
5. (+) Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.

Linear, Quadratic, and Exponential Models★F -LE

Construct and compare linear, quadratic, and exponential models and solve problems

1. Distinguish between situations that can be modeled with linear functions and with exponential functions.
   v. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.
   w. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
   x. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.
2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
3. Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.

4. For exponential models, express as a logarithm the solution to abct = d where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.

Interpret expressions for functions in terms of the situation they model

5. Interpret the parameters in a linear or exponential function in terms of a context.

Trigonometric Functions F-TF

Extend the domain of trigonometric functions using the unit circle

1. Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
2. Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.
3. (+) Use special triangles to determine geometrically the values of sine, cosine, tangent for □/3, □/4 and □/6, and use the unit circle to express the values of sine, cosine, and tangent for □–x, □+x, and 2□–x in terms of their values for x, where x is any real number.
4. (+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.

Model periodic phenomena with trigonometric functions

5. Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.
6. (+) Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.
7. (+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.

Prove and apply trigonometric identities

8. Prove the Pythagorean identity sin2(θ) + cos2(θ) = 1 and use it to find sin(θ), cos(θ), or tan(θ) given sin(θ), cos(θ), or tan(θ) and the quadrant of the angle.
9. (+) Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.

Unit Essential Questions

- How did the pioneers of calculus discover the calculus of infinite series?
- How is it possible to determine if a series converges or diverges?
- How can series be used to approximate functions, integrals, and derivatives?

Unit Enduring Understandings

Students will understand that…

- Converging and diverging series.
- How series can be used to approximate functions, integrals and derviatives.

Unit Objectives

Students will know…

- Geometric Series.
- Representing Functions by Series.
- Differentiation and Integration with series.
- Identifying a Series.
- Constructing a Series.
- Series of sin x and cos x.
- Maclaurin and Taylor Series Combining Taylor Series.
- Table of Maclaurin Series.
- About Taylor Polynomials.
- The Remainder Theorem.
- Remainder Estimation Theorem.
- Euler's Formula.
- Convergence.
- nth-term test.

Unit Objectives

Students will be able to…

- Apply the properties of geometric series.
- Differentiate, integrate, or substitute into a known power series in order to find additional power series representations.
- Use derivatives to find the Maclaurin Series or Taylor series generated by a differentiable function.
- Approximate a function with a Taylor polynomial .
- Analyze the truncation error of a series using graphical methods or the Remainder Estimation Theorem.
- Use Euler's formula to relate the functions sin x, cos x, and e^x .
- Use the nth-term Test, the Direct Comparison Test, and the Ratio Test to determine the convergence or divergence of a series of numbers or the radius of convergence of a power series.
- Use the Integral Test and the Alternating Series Test to determine the convergence or divergence of a series of

• Comparing Nonnegative Series. • Ratio test. • Endpoint Convergence • Integral Test. • Harmonic Series and p-series. • Comparison Test. • Alternating Series. • Absolute and Conditional Convergence. • Intervals of Convergence.	numbers. • Determine the convergence or divergence of p-series, including the harmonic series. • Determine the absolute convergence, conditional convergence, or divergence of a power series at the endpoints of its interval of convergence.

OCEAN COUNTY MATHEMATICS CURRICULUM
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Curriculum development Resources/Instructional Materials/Equipment Needed Teacher Resources:

- Textbook: Calculus Graphical, Numerical, Algebraic

Teacher Notes:

OCEAN COUNTY MATHEMATICS CURRICULUM
SOUTHERN REGIONAL SCHOOL DISTRICT
Unit Overview

Content Area: Mathematics

Unit Title: Applications of the derivatives

Target Course/Grade Level: Calculus II Advanced Placement BC / 12

Unit Summary
Derivatives can be used to solve many extreme value problems and related rate problems.

Primary interdisciplinary connections:
Infused within the unit are connection to the 2009 NJCCCS for Mathematics, Language Arts Literacy and Technology.

21st century themes:
The unit will integrate the 21st Century Life and Career stand 9.1 strands A-D. These strands include: Critical thinking and problem solving, creativity and innovation, collaboration, teamwork and leadership, and cross cultural understanding and interpersonal communication.

Technology connections:
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Content Standards
Interpreting Functions F-IF
Understand the concept of a function and use function notation

25. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If *f* is a function and *x* is an element of its domain, then *f*(*x*) denotes the output of *f* corresponding to the input *x*. The graph of *f* is the graph of the equation *y* = *f*(*x*).
26. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
27. Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. For *example, the Fibonacci sequence is defined recursively by f(0) = f(1) = 1, f(n+1) = f(n) +f(n-1) for n ≥ 1.*

Interpret functions that arise in applications in terms of the context

4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*
5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*★
6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

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9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*

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2. Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.

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   gg. Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. *For example, f(x) =2 x3 or f(x) = (x+1)/(x–1) for x ≠ 1.*

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5. (+) Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.

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   z. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.

   aa. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

3. Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.

4. For exponential models, express as a logarithm the solution to *ab*ct = *d* where *a*, *c*, and *d* are numbers

and the base *b* is 2, 10, or *e*; evaluate the logarithm using technology.

Interpret expressions for functions in terms of the situation they model

5. Interpret the parameters in a linear or exponential function in terms of a context.

Trigonometric Functions F-TF

Extend the domain of trigonometric functions using the unit circle

1. Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
2. Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.
3. (+) Use special triangles to determine geometrically the values of sine, cosine, tangent for □/3, □/4 and □/6, and use the unit circle to express the values of sine, cosine, and tangent for □–*x*, □+*x*, and 2□–*x* in terms of their values for *x*, where *x* is any real number.
4. (+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.

Model periodic phenomena with trigonometric functions

5. Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.
6. (+) Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.
7. (+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.

Prove and apply trigonometric identities

8. Prove the Pythagorean identity sin2(θ) + cos2(θ) = 1 and use it to find sin(θ), cos(θ), or tan(θ) given sin(θ), cos(θ), or tan(θ) and the quadrant of the angle.
9. (+) Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.

Unit Essential Questions

- How can derivatives be used to solve real problems?

Unit Enduring Understandings

Students will understand that…

- Derivatives can be used to solve real world problems.

Unit Objectives

Students will know…

- Applications of Derivatives.
- Extreme Value Problems.
- Linear Approximations.
- Newton's Method.

Unit Objectives

Students will be able to…

- Solve extreme value problems and related rate problems.

OCEAN COUNTY MATHEMATICS CURRICULUM SOUTHERN REGIONAL SCHOOL DISTRICT Evidence of Learning
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Teacher Notes:

OCEAN COUNTY MATHEMATICS CURRICULUM
SOUTHERN REGIONAL SCHOOL DISTRICT
Unit Overview

Content Area: Mathematics

Unit Title: Parametric, Vector and Polar Functions

Target Course/Grade Level: Calculus II Advanced Placement BC / 12

Unit Summary
Very often it is impossible to express a curve in function notation. Therefore, polar notation is necessary to use. This chapter will give students the tools they need to work with the calculus of area, length, and motion, but with polar graphs.

Primary interdisciplinary connections:
Infused within the unit are connection to the 2009 NJCCCS for Mathematics, Language Arts Literacy and Technology.

21st century themes:
The unit will integrate the 21st Century Life and Career stand 9.1 strands A-D. These strands include: Critical thinking and problem solving, creativity and innovation, collaboration, teamwork and leadership, and cross cultural understanding and interpersonal communication.

Technology connections:
For further clarification refer to NJ Class Standard Introductions at http://www.state.nj.us/education/cccs/

Learning Targets

Content Standards
Interpreting Functions F-IF
Understand the concept of a function and use function notation

28. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.
29. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
30. Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. For *example, the Fibonacci sequence is defined recursively by* $f(0) = f(1) = 1$, $f(n+1) = f(n) + f(n-1)$ *for* $n \geq 1$.

Interpret functions that arise in applications in terms of the context

4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*
5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*★
6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

Analyze functions using different representations

7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
    - tt. Graph linear and quadratic functions and show intercepts, maxima, and minima.
    - uu. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.

vv. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.

ww. (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.

xx. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

    s. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.

    t. Use the properties of exponents to interpret expressions for exponential functions. *For example, identify percent rate of change in functions such as y = (1.02)t, y = (0.97)t, y = (1.01)12t, y = (1.2)t/10, and classify them as representing exponential growth or decay.*

9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*

Building Functions F-BF

Build a function that models a relationship between two quantities

1. Write a function that describes a relationship between two quantities.

    bb. Determine an explicit expression, a recursive process, or steps for calculation from a context.

    cc. Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.*

    dd. (+) Compose functions. *For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.*

2. Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.

Build new functions from existing functions

3. Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.*

4. Find inverse functions.

    kk. Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. *For example, f(x) =2 x3 or f(x) = (x+1)/(x–1) for x ≠ 1.*

    ll. (+) Verify by composition that one function is the inverse of another.

    mm. (+) Read values of an inverse function from a graph or a table, given that the function has an inverse.

    nn. (+) Produce an invertible function from a non-invertible function by restricting the domain.

5. (+) Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.

Linear, Quadratic, and Exponential Models★F -LE

Construct and compare linear, quadratic, and exponential models and solve problems

1. Distinguish between situations that can be modeled with linear functions and with exponential functions.

    bb. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.

    cc. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.

    dd. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

3. Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.

4. For exponential models, express as a logarithm the solution to *ab*ct = *d* where *a*, *c*, and *d* are numbers and the base *b* is 2, 10, or *e*; evaluate the logarithm using technology.

Interpret expressions for functions in terms of the situation they model

5. Interpret the parameters in a linear or exponential function in terms of a context.

Trigonometric Functions F-TF

Extend the domain of trigonometric functions using the unit circle

1. Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
2. Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.
3. (+) Use special triangles to determine geometrically the values of sine, cosine, tangent for π/3, π/4 and π/6, and use the unit circle to express the values of sine, cosine, and tangent for π–*x*, π+*x*, and 2π–*x* in terms of their values for *x*, where *x* is any real number.
4. (+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.

Model periodic phenomena with trigonometric functions

5. Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.
6. (+) Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.
7. (+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.

Prove and apply trigonometric identities

8. Prove the Pythagorean identity sin2(θ) + cos2(θ) = 1 and use it to find sin(θ), cos(θ), or tan(θ) given sin(θ), cos(θ), or tan(θ) and the quadrant of the angle.
9. (+) Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.

Unit Essential Questions

- What are the other ways to write equations for curves other than to use function notation?
- What is the calculus that relates to these other forms?

Unit Enduring Understandings

Students will understand that…

- Very often it is impossible to express a curve in function notation. Therefore, polar notation is necessary to use. This chapter will give students the tools they need to work with the calculus of area, length, and motion, but with polar graphs.

Unit Objectives

Students will know…

- Derivatives with Parametric Equations.
- Lengths of a smooth curve using parametric equations.
- Cycloids.
- Surface Area.
- Component form.
- Zero Vector.
- Vector Operations.
- Angle Between Vectors.
- Standard Unit Vectors Planar Curves.
- Limits and Continuity.
- Derivatives and Motion.
- Differentiation Rules.
- IntegralsVectors in the plane.
- Modeling Projectile Motion.
- Polar Coordinates.

Unit Objectives

Students will be able to…

- Find derivatives and second derivatives of parametrically defined functions.
- Calculate lengths of parametrically defined curves and calculate surface areas.
- Represent vectors in the form <a,b> and perform algebraic computations involving vectors.
- Differentiate and integrate vector-valued functions.
- Analyze the motion of a particle in space given its position, velocity, or acceleration as a vector function of time.
- Solve problems involving ideal projectile motion and projectile motion with air resistance.
- Graph polar equations and determine the symmetry of polar graphs.
- Convert Cartesian equations into polar form and vice versa.
- Calculate slopes, length, areas of regions in the plane and surface area determined by polar curves.

• Polar Graphs. • Relating Polar and Cartesian Coordinates. • Area in the plane (polar). • Length of a curve (polar). • Area of a Surface of revolution (polar).	

OCEAN COUNTY MATHEMATICS CURRICULUM
SOUTHERN REGIONAL SCHOOL DISTRICT
Evidence of Learning

Formative Assessments
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- Homework
- Class participation

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- Peer tutoring
- Cooperative learning groups
- Modified assignments
- Differentiated instruction
- Native language texts and native language to English dictionary
- Follow all IEP modifications/504 plan

Curriculum development Resources/Instructional Materials/Equipment Needed Teacher Resources:

- Textbook: Calculus Graphical, Numerical, Algebraic

Teacher Notes:

OCEAN COUNTY MATHEMATICS CURRICULUM
SOUTHERN REGIONAL SCHOOL DISTRICT
Unit Overview

Content Area: Mathematics

Unit Title: Preparation for AP test

Target Course/Grade Level: Calculus II Advanced Placement BC / 12

Unit Summary
This unit will help students pull together all the new ideas they have learned.

Primary interdisciplinary connections:
Infused within the unit are connection to the 2009 NJCCCS for Mathematics, Language Arts Literacy and Technology.

21st century themes:
The unit will integrate the 21st Century Life and Career stand 9.1 strands A-D. These strands include: Critical thinking and problem solving, creativity and innovation, collaboration, teamwork and leadership, and cross cultural understanding and interpersonal communication.

Technology connections:
For further clarification refer to NJ Class Standard Introductions at http://www.state.nj.us/education/cccs/

Learning Targets

Content Standards
Interpreting Functions F-IF
Understand the concept of a function and use function notation

31. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.
32. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
33. Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. For *example, the Fibonacci sequence is defined recursively by* $f(0) = f(1) = 1$, $f(n+1) = f(n) + f(n-1)$ *for* $n \geq 1$.

Interpret functions that arise in applications in terms of the context

4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*
5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*★
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2. Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.

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Unit Essential Questions	**Unit Enduring Understandings** *Students will understand that…*
• How do all the ideas fit together that we have been studying?	• This unit will help students pull together all the new ideas they have learned.
Unit Objectives *Students will know…*	**Unit Objectives** *Students will be able to…*
• L'Hopitals Rule. • Improper Integrals. • Partial Fractions. • Infinite Series. • Parametric, Vector and Polar Functions.	• Use skills learned in the past three units to solve problems.

OCEAN COUNTY MATHEMATICS CURRICULUM
SOUTHERN REGIONAL SCHOOL DISTRICT
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