OCEAN COUNTY MATHEMATICS CURRICULUM
SOUTHERN REGIONAL SCHOOL DISTRICT

Content Area: Mathematics

Course Title: Calculus I Advanced Placement AB

Grade Level: 12

Unit Plan	Pacing Guide
Unit Plan 1 **Prerequisites for Calculus**	**Pacing Guide** **1 week**
Unit Plan 2 **Limits and Continuity**	**Pacing Guide** **2 weeks**
Unit Plan 3 **Derivatives**	**Pacing Guide** **5-6 weeks**
Unit Plan 4 **Applications of Derivatives**	**Pacing Guide** **4 weeks**
Unit Plan 5 **Definite Integrals**	**Pacing Guide** **3-4 weeks**
Unit Plan 6 **Differential Equations and Mathematical Modeling**	**Pacing Guide** **3-4 weeks**
Unit Plan 7 **Define Integrals: Applications**	**Pacing Guide** **2-3 weeks**

Date Created: February 2012

Board Approved on: March 14, 2012

OCEAN COUNTY MATHEMATICS CURRICULUM
SOUTHERN REGIONAL SCHOOL DISTRICT
Unit Overview

Content Area: Mathematics

Unit Title: Prerequisites for Calculus

Target Course/Grade Level: Calculus I Advanced Placement AB / 12

Unit Summary
Students will develop an understanding for the five main topics to be included in the Calculus I AP course. Students will not develop proficiency in any of the topics. It is simply a quick five-day walk through the main topics of the course.

Primary interdisciplinary connections: Infused within the unit are connection to the 2009 NJCCCS for Mathematics, Language Arts Literacy and Technology.

21st century themes:
The unit will integrate the 21st Century Life and Career stand 9.1 strands A-D. These strands include: Critical thinking and problem solving, creativity and innovation, collaboration, teamwork and leadership, and cross cultural understanding and interpersonal communication.

Technology connections:
For further clarification refer to NJ Class Standard Introductions at:
http://www.state.nj.us/education/cccs/

Learning Targets

Content Standards
Interpreting Functions F-IF
Understand the concept of a function and use function notation

1. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.
2. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
3. Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by* $f(0) = f(1) = 1$, $f(n+1) = f(n) + f(n-1)$ *for* $n \geq 1$.

Interpret functions that arise in applications in terms of the context

4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*
5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*
6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

Analyze functions using different representations

7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
    a. Graph linear and quadratic functions and show intercepts, maxima, and minima.
    b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
    c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
    d. (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.
    e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.
8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
    a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.
    b. Use the properties of exponents to interpret expressions for exponential functions. *For example, identify percent rate of change in functions such as y = (1.02)t, y = (0.97)t, y = (1.01)12t, y = (1.2)t/10, and classify them as representing exponential growth or decay.*
9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*

Building Functions F-BF

Build a function that models a relationship between two quantities

1. Write a function that describes a relationship between two quantities.
    a. Determine an explicit expression, a recursive process, or steps for
    b. calculation from a context.
    c. Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.*
    d. (+) Compose functions. *For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.*
2. Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.

Build new functions from existing functions

3. Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.*
4. Find inverse functions.
    a. Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. *For example, f(x) =2 x3 or f(x) = (x+1)/(x–1) for x ≠1.*
    b. (+) Verify by composition that one function is the inverse of another.
    c. (+) Read values of an inverse function from a graph or a table, given that the function has an inverse.
    d. (+) Produce an invertible function from a non-invertible function by restricting the domain.
5. (+) Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.

Linear, Quadratic, and Exponential Models★F -LE

Construct and compare linear, quadratic, and exponential models and solve problems

1. Distinguish between situations that can be modeled with linear functions and with exponential functions.

a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.
b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
3. Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.
4. For exponential models, express as a logarithm the solution to *ab*ct = *d* where *a*, *c*, and *d* are numbers and the base *b* is 2, 10, or *e*; evaluate the logarithm using technology.

Interpret expressions for functions in terms of the situation they model

5. Interpret the parameters in a linear or exponential function in terms of a context.

Trigonometric Functions F-TF

Extend the domain of trigonometric functions using the unit circle

1. Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
2. Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.
3. (+) Use special triangles to determine geometrically the values of sine, cosine, tangent for □/3, □/4 and □/6, and use the unit circle to express the values of sine, cosine, and tangent for □–*x*, □+*x*, and 2□–*x* in terms of their values for *x*, where *x* is any real number.
4. (+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.

Model periodic phenomena with trigonometric functions

5. Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.★
6. (+) Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.
7. (+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.

Prove and apply trigonometric identities

8. Prove the Pythagorean identity sin2(θ) + cos2(θ) = 1 and use it to find sin(θ), cos(θ), or tan(θ) given sin(θ), cos(θ), or tan(θ) and the quadrant of the angle.
9. (+) Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.

Unit Essential Questions • What is Instantaneous Rate of Change? • How can an equation, a graph or a table determine Rate of Change? • What is an Integral of a Function? • How can definite integrals be approximated using Trapezoids? • What is a limit?	**Unit Enduring Understandings** *Students will understand that…* • Students will develop an understanding for the five main topics to be included in the Calculus I AP course. Students will not develop proficiency in any of the topics. It is simply a quick five-day walk through the main topics of the course.
Unit Objectives *Students will know…* • The purpose of this unit to give the students an overview of what calculus is all about. Many students have not seen the big picture prior to studying a class. This walk through will give the students an exposure to what lies ahead in their study of calculus.	**Unit Objectives** *Students will be able to…* • The purpose of this unit to give the students an overview of what calculus is all about. Many students have not seen the big picture prior to studying a class. This walk through will give the students an exposure to what lies ahead in their study of Calculus.

OCEAN COUNTY MATHEMATICS CURRICULUM
SOUTHERN REGIONAL SCHOOL DISTRICT
Evidence of Learning

Formative Assessments
For additional ideas please refer to NJ State DOE classroom application documents:
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- Class participation

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- Chapter/Unit Test
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Modifications (ELLs, Special Education, Gifted and Talented)

- Teacher tutoring
- Peer tutoring
- Cooperative learning groups
- Modified assignments
- Differentiated instruction
- Native language texts and native language to English dictionary
- Follow all IEP modifications/504 plan

Curriculum development Resources/Instructional Materials/Equipment Needed Teacher Resources:

- Textbook: Calculus Graphical, Numerical, Algebraic

Teacher Notes:

OCEAN COUNTY MATHEMATICS CURRICULUM
SOUTHERN REGIONAL SCHOOL DISTRICT
Unit Overview

Content Area: Mathematics

Unit Title: Limits and Continuity

Target Course/Grade Level: Calculus I Advanced Placement AB / 12

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Unit Essential Questions	Unit Enduring Understandings
• What is a limit of a function? • How are limits of a function defined? • How does continuity depend on limits?	*Students will understand that…* • To understand the inter-relationship between limits and continuity of a function. • To apply these ideas to find the slope of a function at a given point.
Unit Objectives *Students will know…* • Average and instantaneous speed. • Definition of a limit. • Properties of a limit. • One-sided and Two-sided limits. • Sandwich Theorem. • Continuity at a point. • Continuous Functions. • Composite Functions. • Intermediate Value Theorem. • Rates of Change.	**Unit Objectives** *Students will be able to…* • Calculate average and instantaneous speeds. • Define and calculate limits of a function and apply properties of a limit. • Use the Sandwich Theorem to calculate limits. • Find and verify end behavior models for various functions. • Calculate limits as the domain values approach infinity (identify vertical and horizontal asymptotes). • Identify intervals where a function is continuous. • Rewrite a removable discontinuity by extending or modifying the function. • Apply the Intermediate Value Theorem and the properties of algebraic combinations and composites of continuous functions. • Apply directly the definition of slope of a curve in order to calculate slopes • Find equations of tangent lines and normal lines to a curve at a given point. • Find average rate of change of a function.

COUNTY MATHEMATICS CURRICULUM
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OCEAN COUNTY MATHEMATICS CURRICULUM
SOUTHERN REGIONAL SCHOOL DISTRICT
Unit Overview

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Unit Title: Derivatives

Target Course/Grade Level: Calculus I Advanced Placement AB / 12

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Unit Essential Questions

- How does one determine the slope of a function at a given point?
- What is differential calculus?
- How does one find the derivative of a function?

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Unit Enduring Understandings

Students will understand that…

- Derivatives can be used to understand the behavior of a function. This unit develops a student's ability to find a derivative using the definition of a derivative. This definition is also based on a limit, which was developed in the previous unit.
- Students must know how to find a derivative for many applications exercises in the next part of the course.

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Unit Objectives

Students will know…

- Definition of a derivative.
- Notation related to derivatives of a function.
- Relationships between graphs of f and f'.
- Graphing the derivative from data.
- When does a derivative fail to exist?
- Differentiability implies local linearity and continuity.
- Finding derivatives graphically and with a calculator.
- Intermediate Value Theorem for Derivatives.
- Rules for derivatives.
- Second and higher order derivatives.
- Motion along a line.
- Derivatives of trigonometric functions.
- Chain Rule.
- Implicit differentiation.
- Derivatives of inverse trigonometric functions.
- Derivatives of exponential/logarithmic functions.

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Unit Objectives

Students will be able to…

- Calculate slopes and derivatives using the definition of a derivative.
- Graph f from f' and f' from the graph of f.
- Find where a function is not differentiable and distinguish between corners, cusps, discontinuities, and vertical tangents.
- Approximate derivatives numerically and graphically.
- Use the rules for derivatives.
- Use derivatives to analyze straight line motion and solve problems with rates of change.
- Use the rules for differentiating trigonometric functions.
- Differentiate composite functions using the chain rule.
- Find slopes of parameterized curves.
- Find derivatives using implicit differentiation.
- Find derivatives of the inverse trigonometric functions.
- Calculate derivatives of exponential and logarithmic functions.

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OCEAN COUNTY MATHEMATICS CURRICULUM
SOUTHERN REGIONAL SCHOOL DISTRICT
Evidence of Learning

Formative Assessments
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http://www.state.nj.us/education/cccs/

- Observation
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- Class participation

Summative Assessments
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- Presentations
- Unit Projects
- Quarterly and Final Exams

Modifications (ELLs, Special Education, Gifted and Talented)

- Teacher tutoring
- Peer tutoring
- Cooperative learning groups
- Modified assignments
- Differentiated instruction
- Native language texts and native language to English dictionary
- Follow all IEP modifications/504 plan

Curriculum development Resources/Instructional Materials/Equipment Needed Teacher Resources:

- Textbook: Calculus Graphical, Numerical, Algebraic

Teacher Notes:

OCEAN COUNTY MATHEMATICS CURRICULUM
SOUTHERN REGIONAL SCHOOL DISTRICT
Unit Overview

Content Area: Mathematics

Unit Title: Applications of Derivatives

Target Course/Grade Level: Calculus I Advanced Placement AB / 12

Unit Summary
Students will develop an understanding for the five main topics to be included in the Calculus I AP course. Students will not develop proficiency in any of the topics. It is simply a quick five-day walk through the main topics of the course.

Primary interdisciplinary connections: Infused within the unit are connection to the 2009 NJCCCS for Mathematics, Language Arts Literacy and Technology.

21st century themes:
The unit will integrate the 21st Century Life and Career stand 9.1 strands A-D. These strands include: Critical thinking and problem solving, creativity and innovation, collaboration, teamwork and leadership, and cross cultural understanding and interpersonal communication.

Technology connections:
For further clarification refer to NJ Class Standard Introductions at:
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Learning Targets

Content Standards

Interpreting Functions F-IF
Understand the concept of a function and use function notation
1. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If *f* is a function and *x* is an element of its domain, then *f*(*x*) denotes the output of *f* corresponding to the input *x*. The graph of *f* is the graph of the equation *y* = *f*(*x*).
2. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
3. Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by f(0) = f(1) = 1, f(n+1) = f(n) + f(n-1) for n ≥ 1.*

Interpret functions that arise in applications in terms of the context
4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*
5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*★
6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval.

Estimate the rate of change from a graph.

Analyze functions using different representations

7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
   a. Graph linear and quadratic functions and show intercepts, maxima, and minima.
   b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
   c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
   d. (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.
   e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.
8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
   a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.
   b. Use the properties of exponents to interpret expressions for exponential functions. *For example, identify percent rate of change in functions such as y = (1.02)t, y = (0.97)t, y = (1.01)12t, y = (1.2)t/10, and classify them as representing exponential growth or decay.*
9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*

Building Functions F-BF

Build a function that models a relationship between two quantities

1. Write a function that describes a relationship between two quantities.
   a. Determine an explicit expression, a recursive process, or steps for calculation from a context.
   b. Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.*
   c. (+) Compose functions. *For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.*
2. Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.

Build new functions from existing functions

3. Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.*
4. Find inverse functions.
   a. Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. *For example, f(x) =2 x3 or f(x) = (x+1)/(x–1) for x ≠ 1.*
   b. (+) Verify by composition that one function is the inverse of another.
   c. (+) Read values of an inverse function from a graph or a table, given that the function has an inverse.
   d. (+) Produce an invertible function from a non-invertible function by restricting the domain.
5. (+) Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.

Linear, Quadratic, and Exponential Models★F -LE

Construct and compare linear, quadratic, and exponential models and solve problems

1. Distinguish between situations that can be modeled with linear functions and with exponential functions.
   a. Prove that linear functions grow by equal differences over equal intervals, and that exponential

functions grow by equal factors over equal intervals.

b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.

c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
3. Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.
4. For exponential models, express as a logarithm the solution to *ab*ct = *d* where *a*, *c*, and *d* are numbers and the base *b* is 2, 10, or *e*; evaluate the logarithm using technology.

Interpret expressions for functions in terms of the situation they model

5. Interpret the parameters in a linear or exponential function in terms of a context.

Trigonometric Functions F-TF

Extend the domain of trigonometric functions using the unit circle

1. Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
2. Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.
3. (+) Use special triangles to determine geometrically the values of sine, cosine, tangent for π/3, π/4 and π/6, and use the unit circle to express the values of sine, cosine, and tangent for π–*x*, π+*x*, and 2π–*x* in terms of their values for *x*, where *x* is any real number.
4. (+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.

Model periodic phenomena with trigonometric functions

5. Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.★
6. (+) Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.
7. (+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.

Prove and apply trigonometric identities

8. Prove the Pythagorean identity sin2(θ) + cos2(θ) = 1 and use it to find sin(θ), cos(θ), or tan(θ) given sin(θ), cos(θ), or tan(θ) and the quadrant of the angle.
9. (+) Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.

Unit Essential Questions	**Unit Enduring Understandings**
• How can derivatives be used to draw conclusions about extreme values of a function and the general shape of a function's graph? • How does a tangent line capture the shape of a curve near a point of tangency? • How can we deduce the rate of change of a function we cannot measure from rates of change we already know? • How can we find a function when we know its derivative?	*Students will understand that…* • Derivatives can be used to gather information about a function.
Unit Objectives *Students will know…* • How to find Local and Absolute extreme values. • Mean Value Theorem. • Increasing and Decreasing Functions. • First Derivative test for Local Extrema. • Concavity. • Second Derivative Test for Local Extrema. • Linear Approximations. • Differentials. • Estimating Change with Differentials. • Sensitivity to Change. • Related Rate Equations. • Simulating Related Motion.	**Unit Objectives** *Students will be able to…* • Determine local or global extreme values of a function. • Apply the Mean Value Theorem and find the intervals on which a function is increasing or decreasing. • Use the First and Second Derivative Tests to determine local extreme values of a function. • Determine the concavity of a function and points of inflection based on the Second Derivative test. • Find linearization's and use Newton's Method to approximate zeros of a function. • Estimate the change in a function using differentials. • Solve related rate problems.

OCEAN COUNTY MATHEMATICS CURRICULUM
SOUTHERN REGIONAL SCHOOL DISTRICT
Evidence of Learning

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Curriculum development Resources/Instructional Materials/Equipment Needed Teacher Resources:

- Textbook: Calculus Graphical, Numerical, Algebraic

Teacher Notes:

OCEAN COUNTY MATHEMATICS CURRICULUM
SOUTHERN REGIONAL SCHOOL DISTRICT
Unit Overview

Content Area: Mathematics

Unit Title: Definite Integrals

Target Course/Grade Level: Calculus I Advanced Placement AB / 12

Unit Summary
Students will develop an understanding for the five main topics to be included in the Calculus I AP course. Students will not develop proficiency in any of the topics. It is simply a quick five-day walk through the main topics of the course.

Primary interdisciplinary connections: Infused within the unit are connection to the 2009 NJCCCS for Mathematics, Language Arts Literacy and Technology.

21st century themes:
The unit will integrate the 21st Century Life and Career stand 9.1 strands A-D. These strands include: Critical thinking and problem solving, creativity and innovation, collaboration, teamwork and leadership, and cross cultural understanding and interpersonal communication.

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2. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
3. Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by* $f(0) = f(1) = 1$, $f(n+1) = f(n) + f(n-1)$ *for* $n \geq$ □ *1.*

Interpret functions that arise in applications in terms of the context

4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*
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Estimate the rate of change from a graph.★

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Interpret expressions for functions in terms of the situation they model

5. Interpret the parameters in a linear or exponential function in terms of a context.

Trigonometric Functions F-TF

Extend the domain of trigonometric functions using the unit circle

1. Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
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3. (+) Use special triangles to determine geometrically the values of sine, cosine, tangent for π/3, π/4 and π/6, and use the unit circle to express the values of sine, cosine, and tangent for π–*x*, π+*x*, and 2π–*x* in terms of their values for *x*, where *x* is any real number.
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9. (+) Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.

Unit Essential Questions	**Unit Enduring Understandings**
• How does instantaneous change accumulate over an interval to produce a function? • How does one find the area under a curve (Integral Calculus)?	*Students will understand that…* • To help students build the relationship between differential and integral calculus (The Fundamental Theorem of Calculus).
Unit Objectives *Students will know…* • Distance traveled. • Rectangular Approximation Method (RAM). • Volume of a sphere. • Riemann Sums. • Terminology and Notation for Integration. • Integrals on a calculator. • Integrals with Discontinuous Functions. • Properties of Definite Integrals. • Average Value of a Function. • Mean Value Theorem for Definite Integrals. • Connecting Differential and Integral Calculus. • Fundamental Theorem of Calculus. • Graphing an integral. • Area connection. • Trapezoidal Rule. • Other approximation methods.	**Unit Objectives** *Students will be able to…* • Approximate the area under the graph of a nonnegative continuous function by using rectangle approximation methods. • Interpret the area under a graph as a net accumulation of a rate of change. • Express the area under a curve as a definite integral and as a limit of Riemann Sums. • Compute the area under a curve using a numerical integration procedure. • Express the area under a curve as a definite integral and as a limit of Riemann Sums. • Compute the area under a curve using a numerical integration procedure. • Apply rules for definite integrals and find the average value of a function over a closed interval. • Apply the Fundamental Theorem of Calculus. • Understand the relationship between the derivative and the definite integral as express in both parts of the Fundamental Theorem of Calculus. • Approximate the definite integral by using Trapezoids and Simpson's Rule.

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Curriculum development Resources/Instructional Materials/Equipment Needed Teacher Resources:

- Textbook: Calculus Graphical, Numerical, Algebraic

Teacher Notes:

OCEAN COUNTY MATHEMATICS CURRICULUM
SOUTHERN REGIONAL SCHOOL DISTRICT
Unit Overview

Content Area: Mathematics

Unit Title: Differential Equations and Mathematical Modeling

Target Course/Grade Level: Calculus I Advanced Placement AB / 12

Unit Summary

Students will develop an understanding for the five main topics to be included in the Calculus I AP course. Students will not develop proficiency in any of the topics. It is simply a quick five-day walk through the main topics of the course.

Primary interdisciplinary connections: Infused within the unit are connection to the 2009 NJCCCS for Mathematics, Language Arts Literacy and Technology.

21st century themes:

The unit will integrate the 21st Century Life and Career stand 9.1 strands A-D. These strands include: Critical thinking and problem solving, creativity and innovation, collaboration, teamwork and leadership, and cross cultural understanding and interpersonal communication.

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    a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.

   b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
   c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.
2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
3. Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.
4. For exponential models, express as a logarithm the solution to *ab*ct = *d* where *a*, *c*, and *d* are numbers and the base *b* is 2, 10, or *e*; evaluate the logarithm using technology.

Interpret expressions for functions in terms of the situation they model

5. Interpret the parameters in a linear or exponential function in terms of a context.

Trigonometric Functions F-TF

Extend the domain of trigonometric functions using the unit circle

1. Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
2. Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.
3. (+) Use special triangles to determine geometrically the values of sine, cosine, tangent for □/3, □/4 and □/6, and use the unit circle to express the values of sine, cosine, and tangent for □–*x*, □+*x*, and 2□–*x* in terms of their values for *x*, where *x* is any real number.
4. (+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.

Model periodic phenomena with trigonometric functions

5. Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.
6. (+) Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.
7. (+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.

Prove and apply trigonometric identities

8. Prove the Pythagorean identity sin2(θ) + cos2(θ) = 1 and use it to find sin(θ), cos(θ), or tan(θ) given sin(θ), cos(θ), or tan(θ) and the quadrant of the angle.
9. (+) Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.

Unit Essential Questions	Unit Enduring Understandings
• How can differential equations be used to predict the behavior of a function? • How can differential equations be analyzed analytically, graphically, and numerically to make predictions?	*Students will understand that…* • To show students how differential calculus can be used to make conclusions about the behavior of a function. • To integrate analytical, graphical and numerical techniques in making predictions about the future status of a substance based on it rate of change.
Unit Objectives *Students will know…* • Solving initial value problems. • Antiderivatives and Indefinite integrals. • Properties of Indefinite integrals. • Power Rule in Integral Form. • Trigonometric Integrands. • Substitution in Indefinite Integrals. • Separable Differential Equations. • Integration by parts. • Law of Exponential Change. • Newton's Law of Cooling.	**Unit Objectives** *Students will be able to…* • Construct antiderivatives using the Fundamental Theorem of Calculus. • Find antiderivatives of polynomials, *ekx* , and selected trigonometric functions of kx, as well as linear combination of functions. • Solve initial value problems of the form *dy* *dx* $f(x)$ • Construct slope fields using technology and interpret slope fields as visualizations of differential equations. • Compute indefinite and definite integrals by the method of substitution. • Solve differential equations of the form *dy* *dxf* (x) • Use integration by parts to evaluate indefinite integrals and definite integrals. • Solve problems involving exponential growth and decay in a variety of applications. • Use Euler's method and the improved Euler's method to find an approximate solution to a differential equation with initial values.

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Formative Assessments
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- Observation
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- Class participation

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- Chapter/Unit Test
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- Unit Projects
- Quarterly and Final Exams

Modifications (ELLs, Special Education, Gifted and Talented)

- Teacher tutoring
- Peer tutoring
- Cooperative learning groups
- Modified assignments
- Differentiated instruction
- Native language texts and native language to English dictionary
- Follow all IEP modifications/504 plan

Curriculum development Resources/Instructional Materials/Equipment Needed Teacher Resources:

- Textbook: Calculus Graphical, Numerical, Algebraic

Teacher Notes:

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Unit Overview

Content Area: Mathematics

Unit Title: Definite Integrals: Applications

Target Course/Grade Level: Calculus I Advanced Placement AB / 12

Unit Summary
Up to this point in each student's mathematics experience they have not had any means of generating area of irregularly shaped figures or the volume of a solid that have irregular shapes. This unit provides students with a means to complete both of these tasks.
Primary interdisciplinary connections: Infused within the unit are connection to the 2009 NJCCCS for Mathematics, Language Arts Literacy and Technology.

21st century themes:
The unit will integrate the 21st Century Life and Career stand 9.1 strands A-D. These strands include: Critical thinking and problem solving, creativity and innovation, collaboration, teamwork and leadership, and cross cultural understanding and interpersonal communication.

Technology connections:
For further clarification refer to NJ Class Standard Introductions at:
http://www.state.nj.us/education/cccs/

Learning Targets

Content Standards
Interpreting Functions F-IF
Understand the concept of a function and use function notation
1. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.
2. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
3. Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by* $f(0) = f(1) = 1$, $f(n+1) = f(n) + f(n-1)$ *for* $n \geq$ □ *1.*

Interpret functions that arise in applications in terms of the context
4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*
5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*
6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

Analyze functions using different representations
7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

a. Graph linear and quadratic functions and show intercepts, maxima, and minima.
b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
d. (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.
e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
   a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.
   b. Use the properties of exponents to interpret expressions for exponential functions. *For example, identify percent rate of change in functions such as y = (1.02)t, y = (0.97)t, y = (1.01)12t, y = (1.2)t/10, and classify them as representing exponential growth or decay.*
9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*

Building Functions F-BF

Build a function that models a relationship between two quantities

1. Write a function that describes a relationship between two quantities.
   a. Determine an explicit expression, a recursive process, or steps for calculation from a context.
   b. Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.*
   c. (+) Compose functions. *For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.*
2. Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.

Build new functions from existing functions

3. Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.*
4. Find inverse functions.
   a. Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. *For example, f(x) =2 x3 or f(x) = (x+1)/(x–1) for x ≠ 1.*
   b. (+) Verify by composition that one function is the inverse of another.
   c. (+) Read values of an inverse function from a graph or a table, given that the function has an inverse.
   d. (+) Produce an invertible function from a non-invertible function by restricting the domain.
5. (+) Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.

Linear, Quadratic, and Exponential Models F -LE

Construct and compare linear, quadratic, and exponential models and solve problems

1. Distinguish between situations that can be modeled with linear functions and with exponential functions.
   a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.
   b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
   c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit

interval relative to another.

2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
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2. Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.
3. (+) Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and use the unit circle to express the values of sine, cosine, and tangent for $\pi-x$, $\pi+x$, and $2\pi-x$ in terms of their values for *x*, where *x* is any real number.
4. (+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.

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Unit Essential Questions	**Unit Enduring Understandings** *Students will understand that…*
• How can an integral be used to solve problems? • How is an integral related to the net change made over time? • How can integrals be applied to finding bounded areas and generated volume?	• To show students how differential calculus can be used in real world applications. • To integrate analytical, graphical and numerical techniques to find the area and volume of irregular shapes.
Unit Objectives *Students will know…* • Linear motion revisited. • Consumption over time. • Net change from data. • Area between curves. • Area enclosed by intersecting curves. • Boundaries with changing functions. • Integrating with respect to y. • Saving time with geometry formulas. • Volume as an integral. • Square cross sections. • Circular cross sections. • Cylindrical shells. • Other cross sections.	**Unit Objectives** *Students will be able to…* • Solve problems in which a rate is integrated to find the net change over time in a variety of applications. • Use integration to calculate area of regions in a plane. • Use integration (by slices or shells) to calculate volumes of solids. • Use integration to calculate surface areas of solids of revolutions.

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